

MAT 1400: End-of-term Exam
April 24, 2019

Name: _____

No calculators, books, or notes may be used for this exam.

1. Let $f(x) = \sqrt{x-2}$, and let $g(x) = 2x^3$.

- (a) What is the domain of f ? _____
- (b) What is the domain of g ? _____
- (c) Write the formula for $f \circ g(x)$. _____
- (d) What is the domain of $f \circ g$? _____
- (e) What is the domain of $\frac{g}{f}$? _____

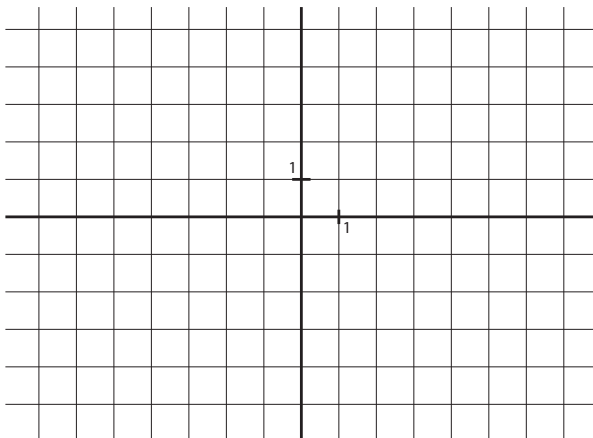
2. Let $h : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $h(x) = (x+2)^2$.

(a) Provide formulas for functions f and g such that $h = f \circ g$.

$g(x) =$

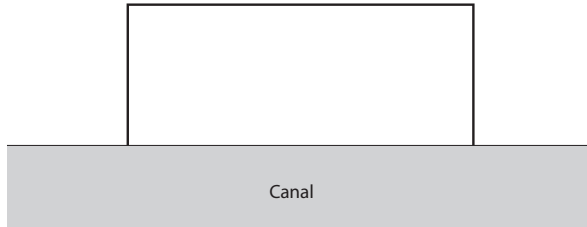
$f(x) =$

(b) Sketch the graph of $y = h(x)$ on the coordinate system below. Your graph should clearly show the x - and y intercepts.

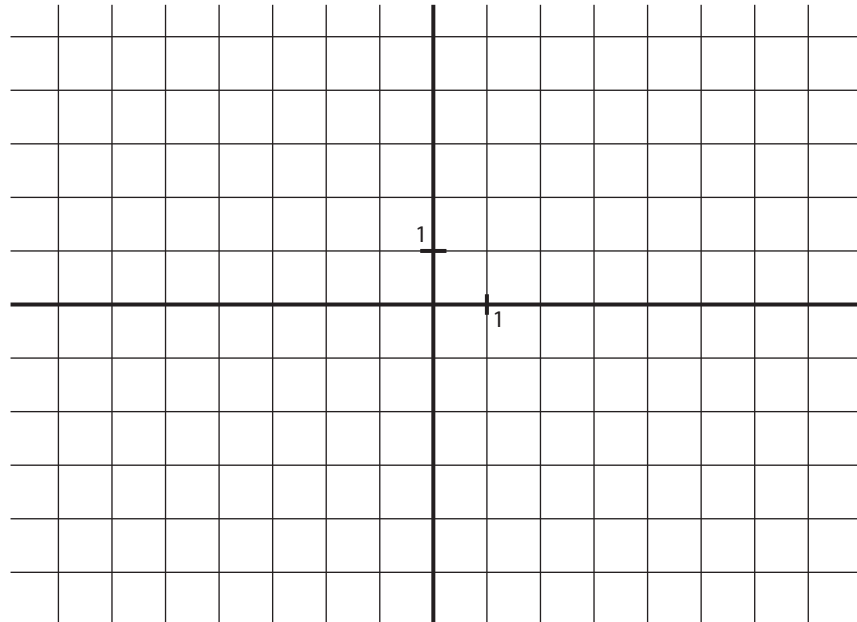


- (c) On what intervals is this function increasing? _____
- (d) On what intervals is this function decreasing? _____
- (e) Does this function have a maximum or minimum value? If so, what is it?
Maximum: Yes No *Circle one.*
If yes, maximum value occurs at $x =$ _____, and the maximum value is $y =$ _____.
Minimum: Yes No *Circle one.*
If yes, minimum value occurs at $x =$ _____, and the minimum value is $y =$ _____.

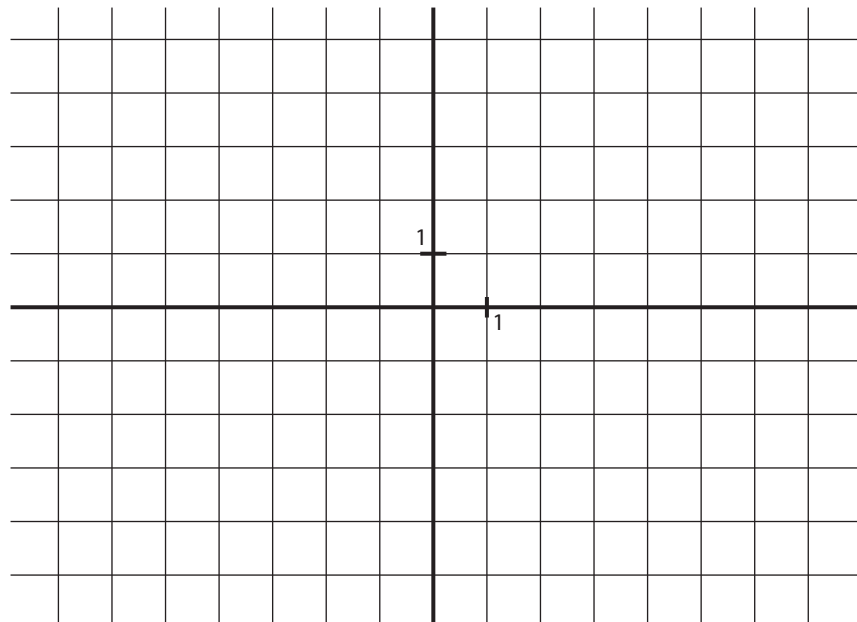
3. A farmer has 60 meters of fence she wants to use to enclose a field that is next to a straight canal. The canal will form one side of the field; fencing is only required on the other three sides. Find the dimensions of the field of largest area that can be enclosed in this way. *Be sure to show your solution process and reasoning!*



4. (a) On the grid below, graph the function $y = f(x) = 2\sqrt{x-1} - 4$. Accurately show the x -intercept on your graph. (Note that $f(5) = 0$.)



- (b) On the grid below, graph the function $y = (x - 1)(x + 1)(x - 2)$, accurately showing the x - and y -intercepts.



5. (a) Use long division of polynomials to write the rational function

$$\frac{x^4 + 2x^3 - 3x + 5}{x^2 - x + 1}$$

as the sum of a polynomial and a proper rational function (that is, one in which the degree of the numerator is less than the degree of the denominator).

$$\frac{x^4 + 2x^3 - 3x + 5}{x^2 - x + 1} =$$

(b) Simplify to $a + bi$ form: $\overline{2 - 3i}$

(c) Simplify to $a + bi$ form: $(2 - 3i)(2 + 3i)$

(d) Simplify to $a + bi$ form: $\frac{2 - 3i}{2 + 3i}$