Chapter 15; Periodic Motion

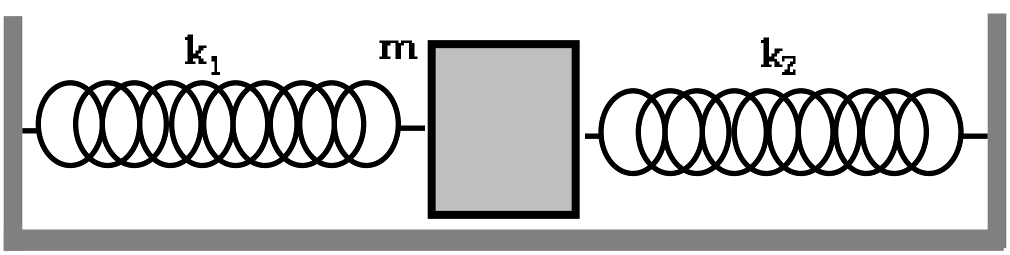
15.9, 11, 12, 15, 20, 25, 31, 34, 35

15.9 This is an excellent one to solve with a spreadsheet:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

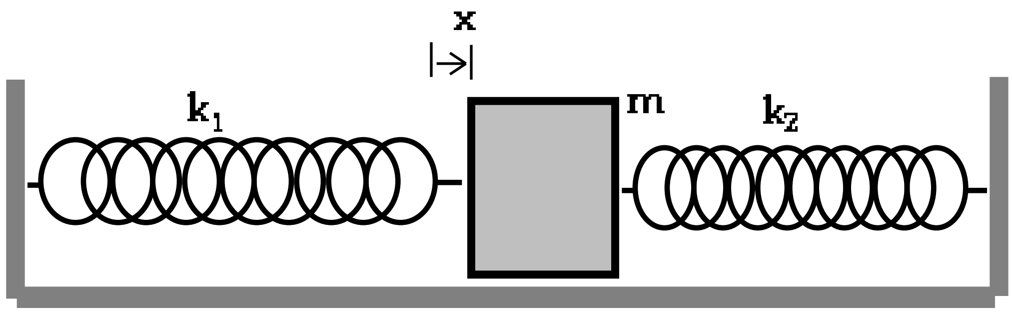
	A	B	C	D	E
1			"=SQRT(C4/B4)/2/3.14159		
2		Mass	Spg Const	Frequency	
3		(kg)	(N/m)	(Hz = 1/s)	
4	a	1	2.5	0.2516	Hz
5	b	2.5	1	0.1007	Hz
6	c	3	7.2	0.2466	Hz
7	d	50	100	0.2251	Hz
8	e	80	200	0.2516	Hz
9	f	100	250	0.2516	Hz
10	g	120	250	0.2297	Hz
11	h	200	500	0.2516	Hz

15.11 $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ **where k in this equation is the "effective spring constant", the spring constant of a <u>single</u> spring that has the same effect as the combination of springs in the diagram.**



With this arrangement, a displacement of x causes a force of

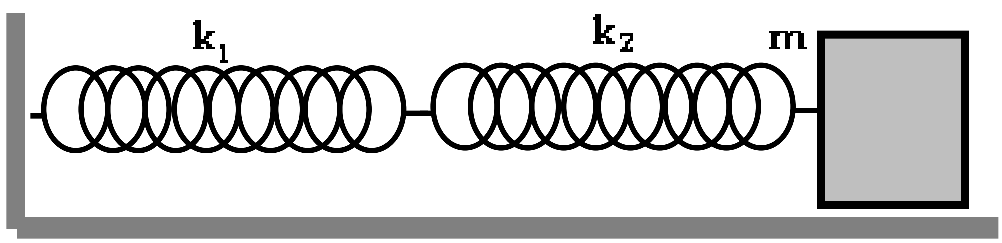
$F_1 = -k_1x$ due to the spring on the left and a force of $F_2 = -k_2x$ due to the spring on the right.



The net force, then, is $F_{net} = F_1 + F_2 = -k_1x - k_2x = -(k_1 + k_2)x$. Since the effective spring constant, k_{eff} is the constant is $F_{net} = -k_{eff}x$, we can see that

$$k_{eff} = k_1 + k_2$$

The next arrangement of springs is a little more "tricky" or more interesting:



When the mass is moved a distance x, spring #1 stretches a distance x_1 and spring #2 stretches a distance x_2 with

$$x = x_1 + x_2$$

That is, the two springs need <u>not</u> be stretched (or compressed) the same amount at all. However, the <u>force</u> exerted by each spring must be the same. That is,

$$F_1 = F_2$$

or

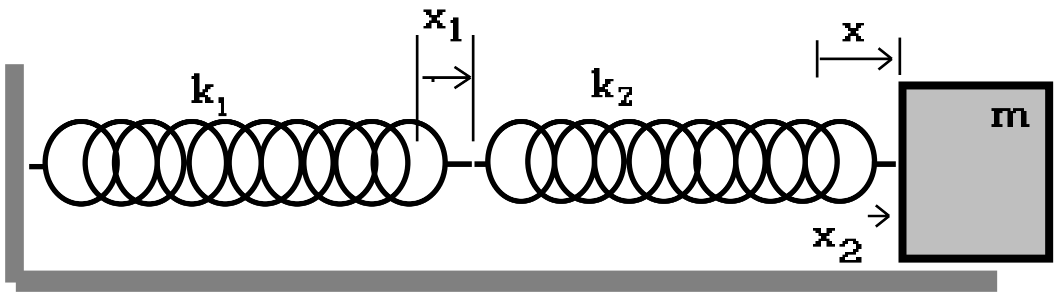
$$k_1 x_1 = k_2 x_2$$

or

$$x_1 = (k_2/k_1)\, x_2$$

or

$$x_2 = (k_1/k_2)\, x_1$$



To determine the "effective spring constant", we must write the force in the form of

$$F = -k_{eff}\,x$$

We have

$$F = -k_1\,x_1$$

and

$$x = x_1 + x_2$$

$$x = x_1 + (k_1/k_2)\,x_1$$

$$x = [\,1 + (k_1/k_2)]\,x_1$$

$$x_1 = \frac{x}{1 + (k_1/k_2)}$$

Therefore,

$$F = k_1\left(\frac{x}{1 + (k_1/k_2)}\right)$$

$$F = k_1\left(\frac{x}{(k_2/k_2) + (k_1/k_2)}\right)$$

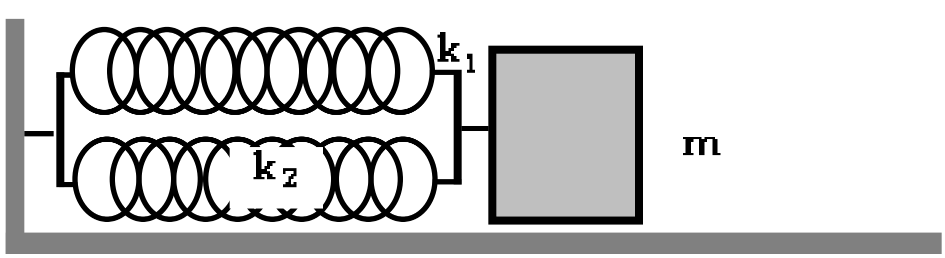
$$F = -\left(\frac{k_1\,k_2}{k_1 + k_2}\right)x$$

Therefore,

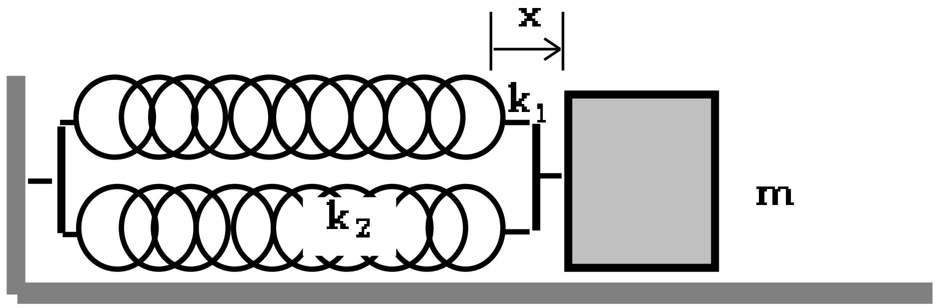
$$k_{eff} = \frac{k_1\,k_2}{k_1 + k_2}$$

which can also be written as

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$$



The third arrangement of springs is actually fairly easy and straightforward. With this arrangement, a displacement of x causes a force of $F_1 = -k_1x$ due to spring #1 and a force of $F_2 = -k_2x$ due to spring #2. The stretch of each spring is the same as the displacement of the mass.



The net force, then, is $F_{net} = F_1 + F_2 = -k_1x - k_2x = -(k_1 + k_2)x$. Since the effective spring constant, k_{eff} is the constant is $F_{net} = -k_{eff}x$, we can see that

$$k_{eff} = k_1 + k_2$$

15.12 $W = (1/2)\ k\ x^2$

$W = (1/2)\ (80\ N/m)\ (0.15\ m)^2$

$W = 0.9\ H$

15.15 $W = PE = (1/2)\ k\ A^2 = (1/2)\ (50\ N/m)\ (0.1\ m)^2 = 0.25\ J$

As the mass moves through equilibrium, x = 0, so it has <u>zero</u> potential energy and its total energy is now KE,

$KE = (1/2)\ m\ v^2 = (1/2)\ (0.2\ kg)\ v^2 = 0.25\ J = E$

$v^2 = 2.5\ m^2/s^2$

$v = 1.58\ m/s$

15.20 $E = PE_{max} = (1/2)\ k\ A^2$

$A = 12\ cm = 0.12\ m$

$E = (1/2)\ (\ k\ )\ (0.12\ m)^2$

$E = KE + PE = \frac{1}{2}\ m\ v^2 + \frac{1}{2}\ k\ x^2$

$E = (1/2)\ m\ (0.2\ m/s^2 + (1/2)\ k\ (0.08\ m)^2 = (1/2)\ (\ k\ )\ (0.12\ m)^2 = E$

$m\ (0.2\ m/s)^2 = k\ (0.12\ m)^2 - k\ (0.08\ m)^2$

$(m/k)(0.2\ m/s)^2 = (0.12\ m)^2 - (0.08\ m)^2$

$(m/k)(0.04)\ (m/s)^2 = (0.0144 - 0.0064)\ m^2 = 0.0080\ m^2$

$(m/k) = 0.2\ (1/s^2)$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$T = 2.8$ sec

15.25 These multiple-data-set problems are good ones to solve with a spreadsheet. This is basically energy conservation,

$$E = (1/2)\, k\, A^2 = (1/2)\, k\, x^2 + (1/2)\, m\, v^2$$

$$k\, A^2 = k\, x^2 + m\, v^2$$

$$m\, v^2 = k\, (A^2 - x^2)$$

$$v^2 = (m/k)\, (A^2 - x^2)$$

$$v = \sqrt{\frac{k}{m}\, (A^2 - x^2)}$$

So this will be the formula we put in the spreadsheet:

	A	B	C	D	E	F
1			'=SQRT((C4/B4)*(D4*D4-E4*E4))			
2		mass	spg cnst	Amplitude	position	velocity
3		(kg)	(N/m)	(m)	(m)	(m/s)
4	a	0.18	3.5	0.2	0.1	0.7638
5	b	0.18	3.5	0.2	0.15	0.5833
6	c	0.28	3.5	0.2	0.18	0.3082
7	d	0.35	5	0.25	0.05	0.9258
8	e	0.35	5.5	0.25	0.1	0.9083
9	f	0.5	5.5	0.25	0.1	0.7599
10	g	0.5	5.5	0.25	0.15	0.6633
11	h	0.5	5.5	0.3	0.15	0.8617

15.31 $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f^2 = (1/4\pi^2)\, \frac{k}{m}$$

$$k = 4\pi^2 f^2\, m$$

$$k = 4\pi^2\, [2\, (1/2)]^2\, (0.5 \text{ kg})$$

$$k = 79 \text{ kg/s}^2$$

$$k = 79 \text{ N/m}$$

15.34 $T = 2.00$ s

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T^2 = 4\pi^2 (l/g)$$

$$l = (1/4\pi^2)T^2 g$$

$$l = 0.993 \text{ m}$$

15.35 $$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T = 2\pi \sqrt{\frac{1.0 \text{ m}}{9.8 \text{ m/s}^2}}$$

$$T = 2.007 \text{ seconds}$$

(That is, this is almost a "seconds pendulum").