PHYsics 1150
Homework, Chapter 14, Thermodynamics
Ch 14: 1, 17, 26, 27, 37, 44, 46, 52, 58

14.1 An ideal gas is sealed in a rigid container at 25°C and 1.0 atm. What will its temperature be when the pressure is incresed to 2.0 atm?
What will its pressure be when the temperature is increased to 50°C

The ideal gas law is

$$PV = nRT$$

Being in a rigid container, the gas' volume does <u>not</u> change; $V = V_o$ = constant.
That means we can use the ideal gas law as

$$T/P = T_o/P_o$$

or

$$T = P\,[T_o/P_o] = [P/P_o]\,T_o$$

Remember, these temperatures must be <u>absolute</u> temperatures,

$$T_o = 25°C = 302\text{ K}$$

$$T = [P/P_o]\,T_o = [2.0\text{ atm}/1.0\text{ atm}]\,[302\text{ K}] = [2]\,[302\text{ K}] = 604\text{ K}$$

$$T = 604\text{ K} = (604 - 273)°C = \mathbf{331°C = T}$$

Or, while V = const, we can write the ideal gas law as

$$P/T = P_o/T_o$$

or

$$P = T\,[P_o/T_o] = [T/T_o]\,P_o$$

Remember, these temperatures must be <u>absolute</u> temperatures,

$$T_o = 25°C = 302\text{ K and } T = 50°C = 327\text{ K}$$

$$P = [T/T_o]\,P_o$$

$$P = [327\text{ K}/302\text{ K}]\,(1\text{ atm})$$

$$\mathbf{P = 1.08\text{ atm}}$$

14.17 Gas at 1.5 atm expands from 2.5 liters to 3.5 liters. How much work is done by the gas?
Work is the area under the curve on a p-V diagram; for constant pressure, this is

$$W = p\,\Delta V$$

$$W = (1.5\text{ atm})(1.0\text{ l})$$

$$W = 1.5\text{ atm-l} = 1.5\text{ l-atm}$$

While this is certainly true, a conversion to units of joules is called for,

$$1\text{ l-atm} = (10^{-3}\text{ m}^3)\,(1.013 \times 10^5\text{ N}/\text{m}^2) = 1.013 \times 10^2\text{ N-m} = 101.3\text{ J}$$

$$W = 1.5\text{ l-atm}\,[101.3\text{ J}/1\text{ l-atm}] = \mathbf{152\text{ J} = W}$$

14.26 Under a constant pressure of 1.5 atm, a gas expands from 2.0 liters to 3.0 liters while 500 J of energy flows into it.
(a) How much work is done by the gas?
(b) What is the change in its internal energy?

Part (a) of this question is **identical** to question 14.17. We already know the answer to that; the work done by the gas is

$$W = 152 \text{ J}$$

As the gas does 152 J of work, it absorbs or receives 500 J of energy. That means its net increase in internal energy is

$$\Delta E_{int} = Q - W = 500 \text{ J} - 152 \text{ J} = 348 \text{ J}$$

$$\Delta E_{int} = 348 \text{ J}$$

14.27 Under a constant pressure of 2.25 atm, a gas is compressed from 2.00 l to 1.50 l while 375 J of energy flows out of the gas.
(a) How much work is done on the gas?
(b) What is the change in its internal energy?

Work is the area under the curve on a p-V diagram; for constant pressure, this is

$$W = p \, \Delta V$$

$$W = (2.25 \text{ atm}) (-0.50 \text{ l})$$

$$W = -1.125 \text{ l-atm}$$

$$W = -1.125 \text{ l-atm} \left[{}^{101.3 \text{ J}}/_{1 \text{ l-atm}} \right] = -114 \text{ J}$$

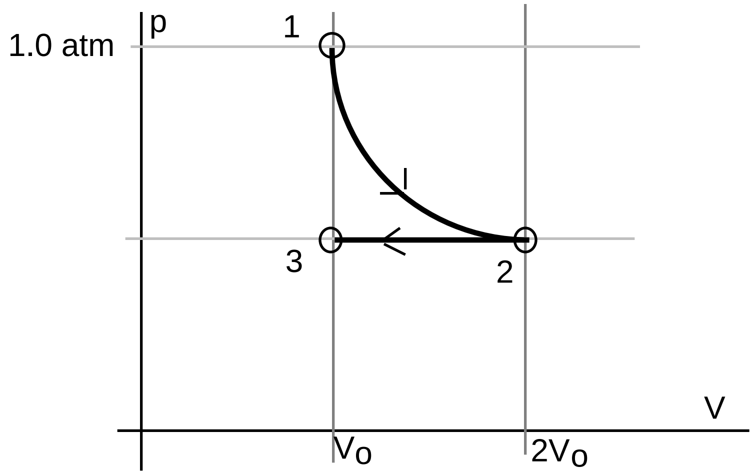
As the gas does $W = -114$ J of work (which is equivalent to having 114 J of work done on it), it also gives up 375 J of energy (which means $Q = -375$ J). That means its net change in internal energy is

$$\Delta E_{int} = Q - W = -375 \text{ J} - (-114 \text{ J})$$

$$\Delta E_{int} = -261 \text{ J}$$

14.37 After 1.0 mol of an ideal gas initially at 0°C and 1.0 atm expands isothermally to twice its initial volume, it is compressed isobarically back to its original volume. What is the net work done by or on the gas? What is its final temperature?

This calls for a p-V diagram.



When the gas expands isothermally from state **1** to state **2**, the work done by the gass is given by Equation 14.17,

$$W_{isotherm} = n\ R\ T\ \ln\ [V_f / V_i]$$

or

$$W_{isotherm} = n\ R\ T\ \ln\ [V_2 / V_1]$$

From the problem statement, we know

$$V_2 = 2\ V_1$$

From this, we can calculate the work done by the gas,

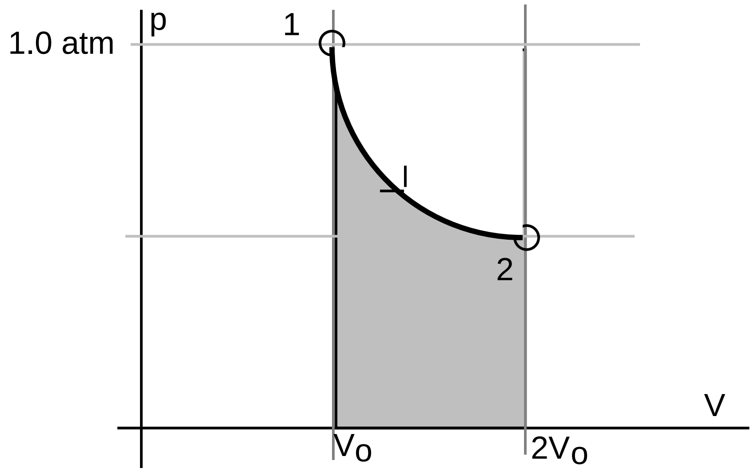
$$W_{isotherm} = (1.0\ mol)(8.314\ {}^{J}/_{mol\text{-}K})\ (273\ K)\ \ln\ [2\ V_1 / V_1]$$

$$W_{isotherm} = (1.0\ mol)(8.314\ {}^{J}/_{mol\text{-}K})\ (273\ K)\ \ln\ [\ 2\ ]$$

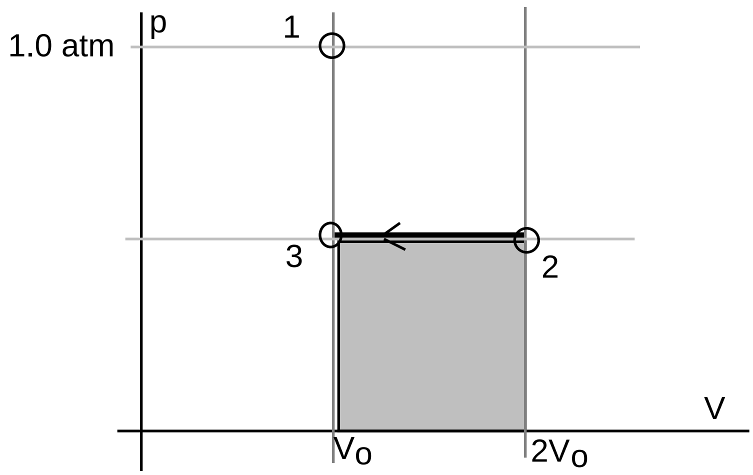
$$W_{isotherm} = (1.0\ mol)(8.314\ {}^{J}/_{mol\text{-}K})\ (273\ K)\ 0.693$$

$$W_{isotherm} = 1{,}573\ J$$

This is the area under the curve from state **1** to state **2**



As the gas is compressed from state **2** to state **3** the work done is, as always, the area under the curve.



$$W_{isobar} = p\,\Delta V = p\,(V_o - 2\,V_o) = -\,p\,V_o$$

We must find p, the pressure for state **2** or state **3**. Since we got to state **2** by an <u>isothermal</u> expansion from state **1**, we know the temperature at state **2** must be the same as at state **1**; $T_1 = T_2 = 273$ K. We can then apply the ideal gas law or, more simply, just use

$$pV = \text{constant} \quad \text{(for constant T)}$$

$$p_2\,V_2 = p_1\,V_1$$

$$p_2 = p_1\,(V_1/V_2) = (1.0\text{ atm})\,(V_1/2\,V_1) = 0.5\text{ atm} = p_2$$

$$p_2 = p_1\,(1/2) = 0.5\text{ atm}$$

To find the work done, $W_{isobar} = -\,pV_o$, we need a numerical value for V_o, the initial volume. We could also call this V_1, the volume at state **1**. To find this initial volume V_o or V_1, we use the ideal gas law,

$$pV = nRT$$

Again, remember that temperatures are often given in degrees Celsius because that is "convenient" but temperatures in the ideal gas law must be <u>absolute</u> temperatures or temperatures on the Kelvin scale!

$$pV = nRT$$

$$(1.0 \text{ atm}) (V_o) = (1.0 \text{ mol}) (8.314 \text{ J/mol-K}) (273 \text{ K})$$

$$(1.0 \text{ atm}) [(1.013 \times 10^5 \text{ N/m}^2)/\text{atm}] (V_o) = (1.0 \text{ mol}) (8.314 \text{ J/mol-K}) (273 \text{ K})$$

$$[1.013 \times 10^5 \text{ N/m}^2] (V_o) = (1.0 \text{ mol})(8.314 \text{ J/mol-K}) (273 \text{ K})$$

$$V_o = 2.24 \times 10^{-2} \text{ m}^3$$

$$V_1 = V_o = 2.24 \times 10^{-2} \text{ m}^3$$

Now we can go back to calculating the work done on/by the gas,

$$W_{isobar} = - p V_o = - (0.5 \text{ atm})(2.24 \times 10^{-2} \text{ m}^3)$$

$$W_{isobar} = - p V_o = - (0.5 \times 1.013 \times 10^5 \text{ N/m}^2)(2.24 \times 10^{-2} \text{ m}^3)$$

$$W_{isobar} = - 1{,}135 \text{ J}$$

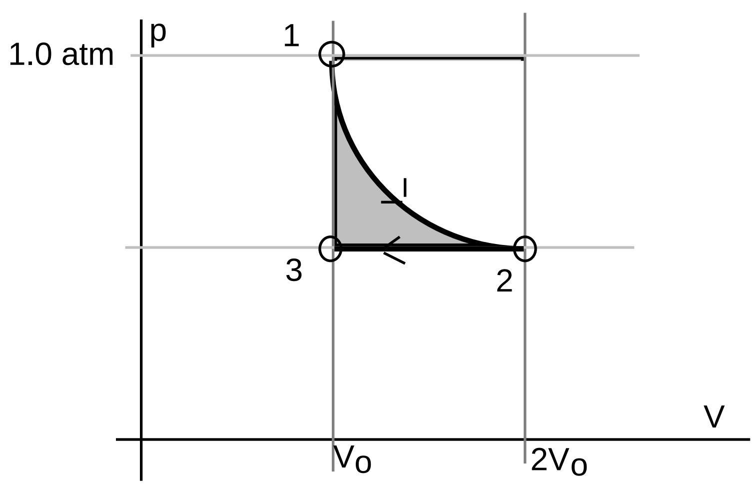
Now we can add these two amounts of work together for the **<u>net</u>** work done.

$$W_{net} = W_{12} + W_{23}$$

$$W_{net} = W_{isotherm} + W_{isobar}$$

$$W_{net} = 1{,}573 \text{ J} + (-1{,}135 \text{ J})$$

$$W_{net} = 438 \text{ J}$$



This **<u>net</u>** work is the area shown on the graph above.

14.44 A heat engine absorbs 6 J from a source of heat and does 2 J of mechanical work. What is its efficiency?

$$\text{Eff} = {}^{W}/_{Q_h}$$
$$\text{Eff} = 2\text{ J} / 6\text{ J} = 0.33$$
$$\text{Eff} = 0.33$$

14.46 A reversible heat engine whose efficience is 15 percent does 180 J of work. How much heat does it absorb from the hot reservoir?

$$\text{Eff} = {}^{W}/_{Q_h}$$
$$Q_h = {}^{W}/_{\text{Eff}}$$
$$Q_h = {}^{180\text{ J}}/_{0.15}$$
$$Q_h = 1{,}200\text{ J}$$

14.52 A Carnot engine operates with a hot reservior of 650°C at 0.30 efficiency. To have 0.35 efficiency, what must the temperature of this erservoir be?

Remember, all these temperatures must be <u>**absolute**</u> temperatures. That means for our initial Carnot engine,

$$T_h = 650°\text{C} = (650 + 273)\text{ K} = 923\text{ K}$$

First, find T_c the temperature of the cold reservoir. That will remain the same.

$$\text{Eff} = 1 - {}^{T_c}/_{T_h} = 0.30$$
$${}^{T_c}/_{T_h} = 1 - 0.30 = 0.70$$
$$T_c = 0.70\, T_h = (0.70)\,(923\text{ K}) = 646\text{ K}$$
$$(\ T_c = 373°\text{C}\ )$$

Now, with this as the cold reservoir (T_c = 646 K), what temperature for the hot reservoir (T_h = ?) will give an efficiency of 0.35?

$$\text{Eff} = 1 - {}^{T_c}/_{T_h} = 0.35$$
$${}^{T_c}/_{T_h} = 1 - 0.35 = 0.65$$
$$T_h = {}^{T_c}/_{0.65}$$
$$T_h = {}^{646\text{ K}}/_{0.65}$$
$$T_h = 993\text{ K}$$
$$(\ T_h = 620°\text{C}\ )$$

14.58 The efficiency of a sasoline engine is 0.55. Estimate the temperature of the combustion gases if the exhaust gases leave the engine at 125°C.

To **<u>estimate</u>** the operation of a real engine, like a gasoline engine, we may treat it as if it were a reversible or Carnot engine. For such a Carnot engine, the efficiency is

$$\text{Eff} = 1 - {}^{T_c}/_{T_h}$$

The exhaust gases leave at the cold temperature of T_c = 125°C = 398 K

$$\text{Eff} = 1 - {}^{T_c}/_{T_h} = 0.55$$

$$^{T_c}/_{T_h} = 1.0 - 0.55 = 0.45$$

$$T_h = {}^{398\text{ K}}/_{0.45}$$

$$T_h = 884\text{ K}$$

$$(\ T_h = 611°\text{C}\ )$$