Chapter 7; Linear Momentum and Collisions

7.4, 9, 23. 30, 33, 34, 36, 43, 48, 53, 56, 63

7.4 What is the momentum of a 1200 kg sedan traveling at 90 km/hr? At what speed must a 3600 kg truck travel to have the same momentum?

First, change the speed to units of m/s,

$$v = 90\ \frac{km}{h}\left[\frac{1000\ m}{km}\right]\left[\frac{h}{3600\ s}\right] = 25\ \frac{m}{s}$$

$$p = m\,v = [1200\ kg]\left[25\ \frac{m}{s}\right] = 30\ 000\ \frac{kg\ m}{s}$$

$$p_{truck} = p_{truck}\ v_{truck}$$

$$v_{truck} = \frac{p_{truck}}{m_{truck}} = \frac{30\ 000\ \frac{kg\ m}{s}}{3600\ kg} = 8.33\ \frac{m}{s}$$

$$v_{truck} = 8.33\ \frac{m}{s}\left[\frac{km}{1000\ m}\right]\left[\frac{3600\ s}{h}\right] = 30\ \frac{km}{h}$$

7.9 A 150 gram baseball initially traveling at 30 m/s is struck by a bat and leaves in the opposite direction at 35 m/s.

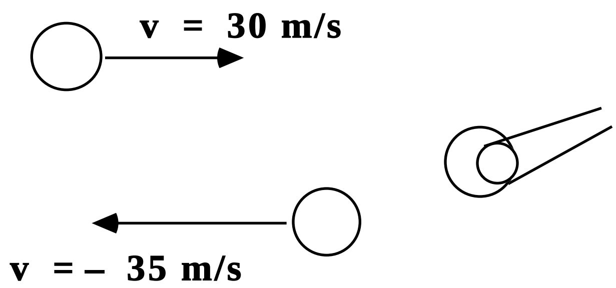
a) What is its change in momentum?

b) Is the change in momentum directed along the initial velocity, the final velocity, or some other direction?

c) What is the impulse delivered by the bat?

d) In hitting a baseball, why is it important to "follow through," that is, keep the bat moving and make a full swing, rather than to stop right after the ball is hit?

Initially, . . .



Finally, . . .

a) $\Delta p = p_f - p_i = (0.150\ kg)(-35\ m/s) - (0.150\ kg)(30\ m/s)$
$= (0.150\ kg)(-65\ m/s) = -9.75\ kg\ m/s$

b) The change in momentum is <u>opposite</u> to the initial momentum and/or along the <u>same</u> direction as the final momentum.

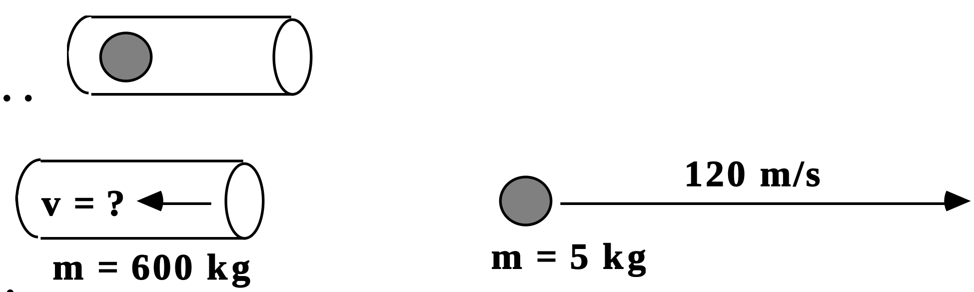
c) "Impulse" is but another word for "change in momentum. Therefore, the impulse is is – 9.75 kg m/s

d) Impulse = Δp = F Δt

By "following through" there is contact for a longer <u>time</u> so that a given force can cause a greater change in momentum. Or a smaller force can cause the same change in momentum.

7.23 A 600 kg cannon fires a 5 kg cannonball with a horizontal muzzle velocity of 120 m/s. What is the recoil velocity of the cannon?

Initially, . . .



Finally, . . .

Initially, with cannon and cannon ball both at rest, the total momentum is <u>zero</u>.

$P_{Tot,\ initial} = 0$

By conservation of momentum, we later expect the total momentum to <u>still</u> be <u>zero</u>. The cannon ball carries momentum to the right so it is positive. The cannon carries momentum to the left so it is negative.

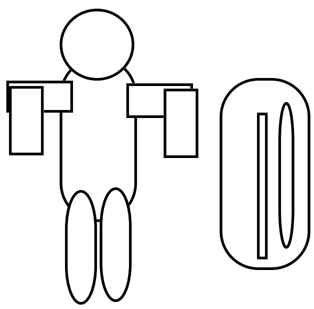
$$P_{Tot,\,final} = p_{ball} + p_{cannon} = (5\text{ kg})(120\ \tfrac{m}{s}) + (600\text{ kg})(v_{cannon}) = 0$$

$$(600\text{ kg})(v_{cannon}) = -(5\text{ kg})(120\ \tfrac{m}{s})$$

$$v_{cannon} = -1\ \tfrac{m}{s}$$

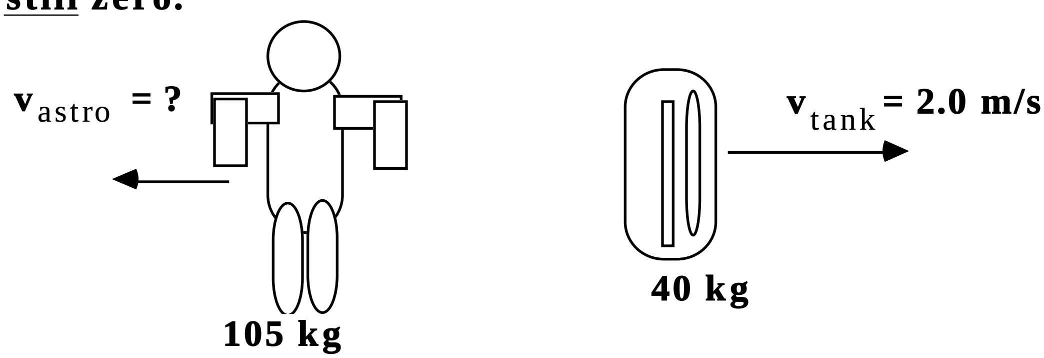
7.30 An astronaut of mass 105 kg carrying an empty oxygen tank of mass 40 kg is stationary relative to a nearby space shuttle. She throws the tank away from herself with a speed of 2 m/s (measured relative to the shuttle). With what velocity relative to the shuttle does the astronaut start to move through space?

Initially, with astronaut and oxygen tank both at rest, the total momentum is zero.



$P_{Tot,\,initial} = 0$

After the astronaut throws the tank, the total momentum is still zero.



$$P_{Tot,\,final} = p_{astro} + p_{tank} = (105\text{ kg})(v_{astro}) + (40\text{ kg})(2.0\ \tfrac{m}{s}) = 0$$

$$(105\text{ kg})(v_{astro}) = -(40\text{ kg})(2.0\ \tfrac{m}{s})$$

$$v_{astro} = -\ 0.76\ \frac{m}{s}$$

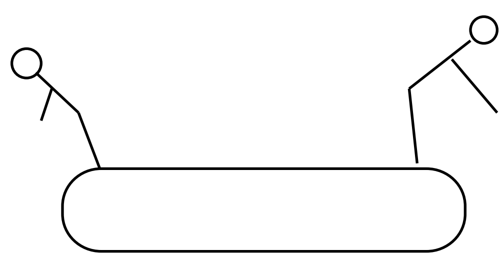
7.33 A 10 000 kg railroad grain car and its load of 3 000 kg of grain coast along a level track at 3.0 m/s. A door is open slightly and lets the grain pour out at a rate of 100 kg/s. What is the speed of the grain car after the grain has all emptied from the car? What has happened to the initial kinetic energy of the railroad car?

As the grain falls out of the car, it does not change the speed of the railroad car and/or the remaining grain in the car. Much of the initial kinetic energy of the grain and car is "lost". As the grain falls from the car it carries kinetic energy with it that is lost to heat as the grain finally comes to rest on the ground.

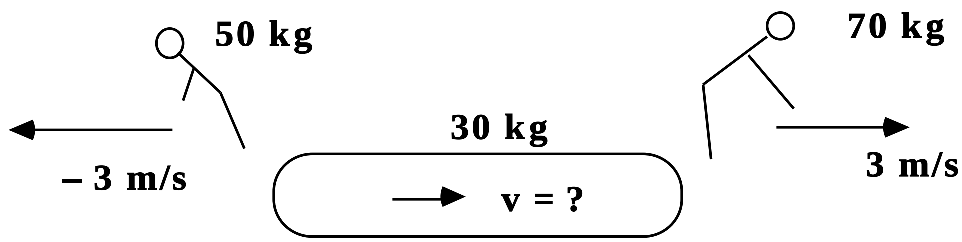
7. 34 An inflated rubber raft of mass 30 kg carries two swimmers of mass 50 kg and 70 kg. The raft and swimmers are initially floating at rest when the swimmers simultaneously dive off from the mid-points of opposite ends of the raft, each with a horizontal velocity of 3 m/s. The 50 kg swimmer dives to the left; the 70 kg swimmer dives to the right. With what speed and in what direction does the raft start to move?

Initially, with both divers and the raft at rest, the total momentum is zero.

$$P_{Tot,\ initial} = 0$$



After the dive, the total momentum is still zero:



$$\mathbf{P_{Tot,\,final} = p_{LeftDiver} + p_{raft} + p_{RightDiver}}$$

$$\mathbf{P_{Tot,\,final} = (50\ kg)(-3\ m/s) + (30\ kg)(v) + (70\ kg)(3\ m/s) = 0}$$

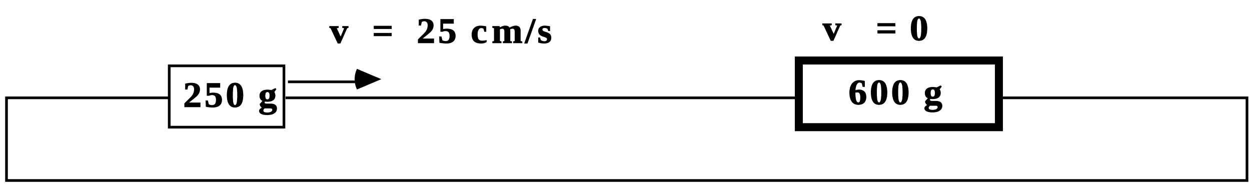
$$\mathbf{-150\ kg\ m/s + (30\ kg)(v) + 210\ kg\ m/s = 0}$$

$$\mathbf{(30\ kg)(v) = -60\ kg\ m/s}$$

$$\mathbf{v = -2\ m/s}$$

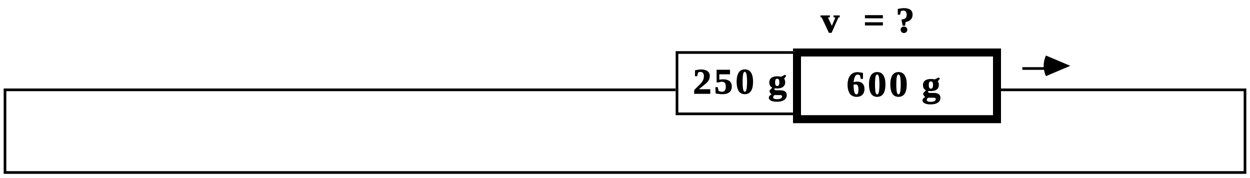
That is, the raft moves to the left at 2 m/s.

7.36 A 600 g glider sits at rest on an air track when it is struck by a 250 g glider traveling at 25 cm/s. The two couple and stick together. What is their velocity?



Initially, $\mathbf{P_{Tot,\,initial} = (250\ g)(25\ cm/s) = 6250\ gm\ cm/s.}$

After the collision,



$P_{Tot, final}$ = (850 g) v = 6250 gm cm/s = $P_{Tot, initial}$

v = 7.35 cm/s

7.43 A 25 g bullet is fired horizontally with a speed of 100 m/s into a 5.0 kg block of wood; the wooden block and bullet start to move off with speed v_f. What is this speed v_f? The block is suspended by cords so that it moves in an arc (but does not rotate) as is illustrated in the figure. What is the height h to which it rises?

$P_{Tot, final}$ = (5.025 kg) v_f = (0.025 kg)(100 m/s) = $P_{Tot, initial}$

v_f = 0.50 m/s

As the block with embedded bullet starts to move, it has KE given by KE = ($^1/_2$) M v^2 = ($^1/_2$) (5.025 kg) (0.50 m/s)^2 = 0.62 J

Relative to the bottom of its swing, its PE is now zero
So its total energy is E = KE + PE = 0.62 J

At the top of its swing, the block (with embedded bullet) momentarily comes to rest so its KE there is zero and all of its energy is now PE = m g h

E = KE + PE = 0 + m g h = m g h

By energy conservation, this final energy must still be 0.62 J

E = m g h = (5.025 kg)(9.8 m/s^2) h = 0.62 J

$h = 0.0126$ m $= 1.26$ cm

7.48 A stationary ^{238}U nucleus with atomic mass 238 x 1.67 x 10^{-27} kg decays by emitting an alpha particle with atomic mass 4 x 1.67 x 10^{-27} kg at a speed of 1.5 x 10^7 m/s. What will be the recoil velocity of the resulting ^{234}Th nucleus?

The "pattern" should be clear by now. Initially, $P_{Tot,\ initial} = 0$

After the radioactive decay, the alpha carries momentum to the right and the thorium nucleus recoils and carries momentum to the left. The total momentum is still zero.

$$P_{Tot,\ final} = (234\ m)(-v) + (4\ m)(1.5 \times 10^7\ m/s) = 0 = P_{Tot,\ initial}$$

where $m = 1.67 \times 10^{-27}$ kg

$$v = \frac{(4)(1.5 \times 10^7\ m/s)}{234}$$

$$v = 2.56 \times 10^5\ m/s$$

7.53 A 600 g glider moves on an air track with a velocity of 10 cm/s to the right while a 400 g glider moves to the left with a velocity of 20 cm/s. The two collide <u>elastically</u>. What is the velocity of each after the collison?

Momentum is <u>always</u> conserved,

$$P_{Tot,fin} = (600\ g)v_{1f} + (400\ g)v_{2f} = (600\ g)(10\ \frac{cm}{s}) + (400\ g)(-20\ \frac{cm}{s})$$

$$= P_{Tot,in}$$

$$(600)v_{1f} + (400)v_{2f} = -2000\ \frac{cm}{s}$$

$$3v_{1f} + 2v_{2f} = -10 \ \frac{cm}{s}$$

or, if we drop the units, this becomes

$$3v_{1f} + 2v_{2f} = -10$$

This is <u>one</u> equation with <u>two</u> unknows, so we must find additional information. Since this is an <u>elastic</u> collision, we <u>also</u> know that the Kinetic Energy is conserved.

$$KE_{Tot,fin} = (^1/_2)(600\ g)v_{1f}{}^2 + (^1/_2)(400\ g)v_{2f}{}^2 =$$

$$= (^1/_2)(600\ g)(10\ cm/s)^2 + (^1/_2)(400\ g)(-20\ cm/s)^2 = KE_{Tot,in}$$

$$(600)v_{1f}{}^2 + (400)v_{2f}{}^2 = (600)(10)^2 + (400)(-20)^2$$

$$3\ v_{1f}{}^2 + 2\ v_{2f}{}^2 = 3\ (10)^2 + 2(-20)^2$$

$$3\ v_{1f}{}^2 + 2\ v_{2f}{}^2 = 1100$$

Now we have <u>two</u> equations in <u>two</u> unknowns. We can solve for v_{2f} from the earlier one and substitute it into this equation,

$$3v_{1f} + 2v_{2f} = -10$$

$$2v_{2f} = -3v_{1f} - 10$$

$$v_{2f} = -1.5v_{1f} - 5$$

$$3\ v_{1f}{}^2 + 2\ [-1.5v_{1f} - 5]^2 = 1100$$

$$3\ v_{1f}{}^2 + 2\ [2.25v_{1f}{}^2 + 15\ v_{1f} + 25] = 1100$$

$$3\ v_{1f}{}^2 + 4.5v_{1f}{}^2 + 30\ v_{1f} + 50 = 1100$$

$$7.5v_{1f}{}^2 + 30\ v_{1f} - 1050 = 0$$

$$v_{1f} = \frac{-30 \pm \sqrt{30^2 - 4\ (7.5)\ (-1050)}}{2\ (7.5)}$$

$$v_{1f} = \frac{-30 \pm \sqrt{900 + 31\ 500}}{2\ (7.5)}$$

$$v_{1f} = \frac{-30 \pm \sqrt{32\ 400}}{15}$$

$$v_{1f} = \frac{-30 \pm 180}{15}$$

There are two possible solutions, due to the ± in the equation;

Either $v_{1f}(1) = \frac{-30 - 180}{15}$ **or** $v_{1f}(2) = \frac{-30 + 180}{15}$

$v_{1f}(1) = -14\ \frac{cm}{s}$ **or** $v_{1f}(2) = 10\ \frac{cm}{s}$

Each of these provides a possible solution for v_{2f}

$v_{2f}(1) = — 1.5v_{1f} - 5$ $v_{2f}(2) = — 1.5v_{1f} - 5$

$v_{2f}(1) = — 1.5(-14) - 5$ $v_{2f}(2) = — 1.5(10) - 5$

$v_{2f}(1) = 21 - 5$ $v_{2f}2) = — 15 - 5$

$v_{2f}(1) = 16$ $v_{2f}2) = — 20$

$v_{2f}(1) = 16\frac{cm}{s}$ $v_{2f}2) = — 20\frac{cm}{s}$

That is, either

$v_{1f}(1) = -14\ \frac{cm}{s}$ $v_{1f}(2) = 10\ \frac{cm}{s}$

$v_{2f}(1) = 16\frac{cm}{s}$ $v_{2f}(2) = — 20\frac{cm}{s}$

The second set is somewhat interesting for

$v_{1f}(2) = 10\ \frac{cm}{s}$ **and** $v_{2f}(2) = — 20\frac{cm}{s}$ **are the <u>initial</u> conditions repeated. This solution is as if the collision had <u>not</u> occured! The solution we seek—the real, physical solution—is the first set,**

$v_{1f}(1) = -14\ \frac{cm}{s}$ **and** $v_{2f}(1) = 16\frac{cm}{s}$

7.56 Below are eight sets of initial conditions for perfectly elastic collisions on an air track. Glider number two is at rest in each case. Calculate the velocity of glider number two in each case and rank them in order of increasing velocity of glider number one.

	mass of glider #1	velocity of glider #1	mass of glider #2	velocity of glider #2
A	300 g	10 cm/s	600 g	0
B	300 g	10 cm/s	300 g	0
C	300 g	10 cm/s	200 g	0
D	600 g	15 cm/s	200 g	0
E	600 g	20 cm/s	250 g	0
F	300 g	25 cm/s	350 g	0
G	250 g	28 cm/s	375 g	0
H	375 g	28 cm/s	250 g	0

Equation 7.13 in the text gives the velocity v_{2f} for just such a case, $v_{2f} = \frac{2\ m_1}{m_1 + m_2} v_{1i}$. Therefore,

$$v_{2f}(A) = \frac{2\ (300\ g)}{300\ g + 600\ g}\ (10\ \frac{cm}{s}) = 6.67\ \frac{cm}{s}$$

$$v_{2f}(B) = \frac{2\ (300\ g)}{300\ g + 300\ g}\ (10\ \frac{cm}{s}) = 10\ \frac{cm}{s}$$

$$v_{2f}(C) = \frac{2\ (300\ g)}{300\ g + 200\ g}\ (10\ \frac{cm}{s}) = 12\ \frac{cm}{s}$$

$$v_{2f}(D) = \frac{2\ (600\ g)}{600\ g + 200\ g}\ (15\ \frac{cm}{s}) = 22.5\ \frac{cm}{s}$$

$$v_{2f}(E) = \frac{2\ (600\ g)}{600\ g + 250\ g}\ (20\ \frac{cm}{s}) = 28.4\ \frac{cm}{s}$$

$$v_{2f}(F) = \frac{2\ (300\ g)}{300\ g + 350\ g}\ (25\ \frac{cm}{s}) = 23\ \frac{cm}{s}$$

$$v_{2f}(G) = \frac{2\ (250\ g)}{250\ g + 375\ g}\ (28\ \frac{cm}{s}) = 22.4\ \frac{cm}{s}$$

$$v_{2f}(H) = \frac{2\ (375\ g)}{375\ g\ +\ 250\ g}\ (28\ \frac{cm}{s}) = 33.6\ \frac{cm}{s}$$

7.63 Exhaust gases leave a certain rocket with a speed of 300 m/s. What must be the mass flow rate in order to lift the 50 000 kg rocket?

$$F_{thrust} = \frac{\Delta m}{\Delta t}\ u = M\ g = Wt$$

$$\frac{\Delta m}{\Delta t} = \frac{M\ g}{u}$$

$$\frac{\Delta m}{\Delta t} = \frac{(50\ 000\ kg)\ (9.8\ m/s^2)}{300\ m/s}$$

$$\frac{\Delta m}{\Delta t} = 1{,}633\ \frac{kg}{s}$$