PHY 1150 Doug Davis Chapter 5, Circular Motion and Gravity 1, 6, 10, 21, 24, 30, 34, 41, 44, 47, 49, 60

5.1 A horse moves with a tangential speed of 1.9 m/s when it is 8.5 m from the center of a carousel. Calculate its centripetal acceleration. If a 70-kg person sits on the horse, what is the net force on the rider?

$$a_{c} = v^{2}/r = \frac{(1.9 \text{ m/s})^{2}}{8.5 \text{ m}} = 0.42 \text{ m/s}^{2}$$

 $F_{c} = m(v^{2}/r) = (70 \text{ kg})(0.42 \text{ m/s}^{2}) = 29.7 \text{ N}$

5.6 A coin sits 15.0 cm from the center of a variable-speed turntable. The coin remains in place as the speed of the turntable increases until it reaches a rate of 60 revolutions per second and then it starts to slide. What is the coefficient of friction between the coin and the surface?

First, find the <u>linear speed</u> coin and its centripetal accleration.

$$\mathbf{v} = \frac{2 \text{ rev } \mathbf{C} \ 2}{\sec \text{ rev } \mathbf{C}} = 60 \quad \frac{\mathbf{cm}}{\sec} = 188.5 \frac{\mathbf{cm}}{\sec}$$
$$\mathbf{a}_{\mathbf{c}} = \frac{\mathbf{v}^2}{\mathbf{r}} = \frac{(188.5 \text{ cm/s})^2}{15 \text{ cm}} = 2,369 \frac{\mathbf{cm}}{\mathbf{s}^2} = 23.7 \frac{\mathbf{m}}{\mathbf{s}^2}$$

To have this acceleration, the <u>net</u> force must be

 $F_{net} = F_c = m v^2 / r = m (23.7 m / s^2)$

Since we do not know the mass of the coin, we will put into the equation simply as "m" and expect that it will later "drop out".



Now apply $\Sigma \mathbf{F} = \mathbf{m} \mathbf{a}$

 $\Sigma \mathbf{F}_{\mathbf{v}} = \mathbf{m} \mathbf{a}_{\mathbf{v}}$

 $\Sigma \mathbf{F}_{\mathbf{x}} = \mathbf{m} \mathbf{a}_{\mathbf{x}}$

 $\sum \mathbf{F}_{\mathbf{y}} = \mathbf{F}_{\mathbf{N}} - \mathbf{mg} = \mathbf{0} \qquad -\mathbf{F}_{\mathbf{f}} = -\mathbf{m} \mathbf{v}^2 / \mathbf{r}$ $\mathbf{F}_{\mathbf{N}} = \mathbf{mg} \qquad \mathbf{F}_{\mathbf{f}} = \mathbf{\mu} \mathbf{F}_{\mathbf{N}}$ $-\mathbf{\mu} \mathbf{F}_{\mathbf{N}} = -\mathbf{m} \mathbf{v}^2 / \mathbf{r}$ $\mathbf{\mu} \mathbf{F}_{\mathbf{N}} = \mathbf{m} \mathbf{v}^2 / \mathbf{r}$ $\mathbf{\mu} \mathbf{mg} = \mathbf{m} \mathbf{v}^2 / \mathbf{r}$ $\mathbf{\mu} \mathbf{g} = \mathbf{v}^2 / \mathbf{r}$ $\mathbf{\mu} = (\mathbf{188.5} \text{ cm/s})^2 / [(\mathbf{980} \text{ cm/s}^2)(\mathbf{15} \text{ cm})]$ $\mathbf{\mu} = 2.41$

 μ = 2.41 is a fairly <u>large</u> coefficient of friction; that is, there must be rubber or something like that on the bottom of the coin.

5.10 A 1200-kg car makes a curve on a flat road of radius 60 m at a speed of 15 m/s. Will the car be able to make the turn if a) the pavement is dry and has a coefficient of friction of 0.65?b) there is oil on the pavement and the coefficient of friction is only 0.20?

The free body diagram for this looks much like the free body diagram for problem 5.9 above. The <u>friction force</u> must provide the centripetal acceleration. Being on a <u>flat</u> curve, the normal force happens to equal the weight; $F_N = m g$.

Now apply Σ F = m a Σ F_y = m a_{yy} Σ F_y = F_N - mg = 0 F_N = mg $\mu = (15 \text{ m/s})^2 / [(9.80 \text{ m/s}^2)(60 \text{ m})]$ $\mu = 0.38$ Σ F_x = m a_x - F_f = -m v² / r μ mg = m v² / r μ g = v² / r $\mu = 0.38$

 μ = 0.38 is the <u>minimum</u> value of the coefficient of friction that is necessary to provide enough friction force to equal the centripetal force required to allow the car to make the curve.

Therefore, ...

a) Yes, the car can make the curve when μ =0.65.

b) No, the car will <u>not</u> make the curve when $\mu = 0.20$.

5.21 The table below lists data for six banked highway curves—giving the radius of each curve and the angle at which the curve is banked. Find the maximum speed at which these curves may be traveled without depending upon friction and rank them from slowest to fastest

| Curve: | Α | B | С | D | Ε | F |
|------------|-----------|-----------|-------|-----------|------------|-------|
| radius | 330 m | 350 m | 280 m | 420 m | 180 m | 250 m |
| bank angle | 5° | 3° | 11° | 5° | 8 ° | 10° |

For calculations that are going to be repeated, it is best to solve the problem symbolically once and then plug particular values into the resulting solution. A spreadsheet is a convenient way to carry out this final calculation.

We already know

$$\tan \theta = \mathbf{v}^2 / \mathbf{r}$$

or

| $\mathbf{v}^2 = \mathbf{r} \mathbf{tan} \theta$ | | | | | | | | |
|---|---|--------|------------|---------|----------|--|--|--|
| | A | В | С | D | E | | | |
| 1 | | radius | bank angle | V | V | | | |
| 2 | | (m) | (degrees) | (m/s) | (km/h) | | | |
| 3 | А | 330 | 5 | 5.4 | 19.3 | | | |
| 4 | В | 350 | 3 | 4.3 | 15.4 | | | |
| 5 | С | 280 | 11 | 7.4 | 26.6 | | | |
| 6 | D | 420 | 5 | 6.1 | 21.8 | | | |
| 7 | E | 180 | 8 | 5.0 | 18.1 | | | |
| 8 | F | 250 | 10 | 6.6 | 23.9 | | | |

5.24 A 60-kg pilot comes out of a dive by flying along an arc of radius 800 m with a speed at the bottom of 120 km/h. What is her apparent weight?



v = 120 km/hr [1000 m/km] [hr/3600 s] = 33.3 m/s

5.30 A car of mass 1000 kg goes over the crest of a hill whose radius of curvature is 40 m measured in a vertical plane.

(a) What is the force of the car on the road surface if the car's speed is 15 m/s?

(b) Calculate the magnitude and direction of the necessary force between car and road if the speed at the top of the crest is 20 m/s. Explain your answer.



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F_{net} = mg - F_N = mv^2/r = F_c
F_N = mg - mv^2/r
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 $F_{N} = m [g - v^{2}/r]$ a) v = 15 m/s $F_{N} = (1000 \text{ kg}) [(9.8 \text{ m/s}^{2}) - \frac{(15 \text{ m/s})^{2}}{40 \text{ m}}]$ $F_{N} = (1000 \text{ kg}) [9.8 - 5.6] \text{ m/s}^{2}$ $F_{N} = (1000 \text{ kg}) [4.2] \text{ m/s}^{2}$ $F_{N} = 4,200 \text{ N} = 4.2 \text{ kN}$ b) v = 20 m/s $F_{N} = (1000 \text{ kg}) [(9.8 \text{ m/s}^{2}) - (20 \text{ m/s})^{2}/(40 \text{ m})]$ $F_{N} = (1000 \text{ kg}) [9.8 - 10] \text{ m/s}^{2}$ $F_{N} = (1000 \text{ kg}) [-0.2] \text{ m/s}^{2}$ $F_{N} = -200 \text{ N}$

Of course, the road can <u>not</u> exert a negative (meaning <u>down</u>ward) normal force. Therefore, the car looses contact with the road!

5.34 In the ROTOR ride on a midway, passengers are pressed against the inside vertical wall of a rotating drum 2.5 m in radius. This drum takes 2.0 seconds to make a complete rotation; that is, it rotates at 30 rpm (revolutions/minute).

(a) Viewed from an Earth-based frame of reference outside the drum, what are the magnitude and direction of the force of the wall on a rider whose mass is 70 kg?

Initially the rider's weight is supported by the floor. After the ROTOR is at full speed the floor is removed.

(b) If the coefficient of static friction between the passenger and the wall is 0.6, will the passenger slip or be held pressed against the vertical wall?

(c) Describe the situations—including the forces—from the reference frame of the passenger.



The normal force, F_N , provides the centripetal force, $F_c = m v_2 / r$.

 $\mathbf{v} = \frac{\Delta \mathbf{s}}{\Delta \mathbf{t}} = \frac{2}{2} \frac{\mathbf{r}}{\sec} = \frac{2}{2} \frac{(2.5 \text{ m})}{2 \sec} = \mathbf{7.8 m/s}$

$$\mathbf{F}_{N} = \mathbf{F}_{c} = \mathbf{m} \frac{\mathbf{v}^{2}}{\mathbf{r}} = (70 \text{ kg}) \frac{(7.8 \text{ m/s})^{2}}{2.5 \text{ m}} = 1,730 \text{ N}$$

For comparison, the rider's weight is $w = mg = (70 \text{ kg})(9.8 \text{ m/s}^2) = 686 \text{ N}$ so the normal force is about two and a half times as large. The normal force is directed <u>inward</u>, toward the center of the circle.

Now the floor of the ROTOR is moved down and the friction force, pointing up, must equal the weight, pointing down, to keep the rider from sliding down along with the floor.

The maximum value of the friction force is

 $F_{f,max} = \mu F_N = (0.6) (1,730 N) = 1,040 N$

The maximum friction force is greater than the weight. Therefore, the actual friction force will just equal the weight and the passenger will remain in place on the wall of the ROTOR. From inside the ROTOR, from the noninertial reference frame, the passenger will feel pushed to the outside by a "centrifugal force" to which the wall responds with the normal force. Because there is a normal force, there can be a friction force and all the rest is the same.

5.41 Calculate the mass of Jupiter, given that its moon Callisto has a mean orbital radius of 1.88 x 10⁶ km and an orbital period of 16 days, 16.54 hours.

The force of gravity provides the centripetal force to keep *Callisto* in its orbit.

$$\mathbf{F}_{\mathbf{g}} = \mathbf{G} \mathbf{M}_{\mathbf{J}} \mathbf{m} / \mathbf{r}^2 = \mathbf{m} \mathbf{v}^2 / \mathbf{r} = \mathbf{F}_{\mathbf{c}}$$
$$\mathbf{M}_{\mathbf{J}} = \frac{\mathbf{r} \mathbf{v}^2}{\mathbf{G}}$$

We must find the linear speed of Callisto.

$$\begin{split} \mathbf{v} &= \frac{2}{T} \mathbf{r} \\ \mathbf{T} &= \mathbf{16} \text{ d}, \mathbf{16.54} \text{ h} = [(\mathbf{16})(\mathbf{24}) + \mathbf{16.54}] \text{ h} = 400.54 \text{ h} \\ \mathbf{T} &= 400.54 \text{ h} [\mathbf{3600 \ s/h}] = \mathbf{1.44} \times \mathbf{10^6} \text{ s} \\ \mathbf{v} &= \frac{2}{(\mathbf{1.88} \times \mathbf{10^6} \text{ km})}{\mathbf{1.44} \times \mathbf{10^6} \text{ s}} = \frac{2}{(\mathbf{1.88} \times \mathbf{10^9} \text{ m})}{\mathbf{1.44} \times \mathbf{10^6} \text{ s}} = \mathbf{8.2} \times \mathbf{10^3} \text{ m/s} \\ \mathbf{M}_J &= \frac{\mathbf{r} \ \mathbf{v}^2}{\mathbf{G}} = \frac{(\mathbf{1.88} \times \mathbf{10^9} \text{ m}) (\mathbf{8.2} \times \mathbf{10^3} \text{ m/s})^2}{\mathbf{6.67} \times \mathbf{10^{-11}} \text{ N} \text{ m}^2/\text{kg}^2} \\ \mathbf{M}_J &= \mathbf{1.9} \times \mathbf{10^{27}} \text{ kg} \end{split}$$

5.44 Starting with the moon's period of 27.3 days, calculate the radius of its orbit.

The gravitational force between Earth and our moon provides the centripetal force,

$$\mathbf{F_g} = \frac{G \frac{M_E m}{r^2}}{r^2} = \mathbf{m} \frac{\frac{v^2}{r}}{r} = \mathbf{F_c}$$

$$G \frac{M_E m}{r^2} = \mathbf{m} \frac{v^2}{r}$$

Don't try to solve for the radius immediately for we know the velocity only in terms of the radius,

$$v = \frac{2}{T} \frac{r}{T}$$

$$\frac{v^2}{r} = \left(\frac{2}{T}\right)^2 \frac{1}{r} = \frac{4}{T^2} \frac{2}{r}$$

$$G \frac{M_E m}{r^2} = m \frac{v^2}{r} = m \frac{4}{T^2} \frac{2}{r^2}$$

$$T = 27.3 \text{ da} (24 \text{ h/da}) (3600 \text{ s/h}) = 2.36 \text{ x} 10^6 \text{ s}$$

$$r^3 = \frac{G M_E T^2}{4^2}$$

$$r^3 = \frac{(6.67 \text{ x} 10^{-11}) (5.98 \text{ x} 10^{24}) (2.36 \text{ x} 10^6)^2}{4^2}$$

$$r^3 = r^3 = 5.627 \text{ x} 10^{25} \text{ m}_3$$

$$r = 3.83 \text{ x} 10^8 \text{ m}$$

$$r = 3.83 \text{ x} 10^5 \text{ km}$$

5.47 The acceleration of a falling body near Earth's surface, at a distance R from Earth's center, is 9.80 m/s².

(a) Use a suitable proportion to calculate the acceleration toward Earth of a falling body that is 60 R from Earth's center.
(b) Our moon is in an orbit of radius 60 R, with a period of revolution of 27.26 days. Show, as did Sir Isaac Newton, that the centripetal acceleration of the moon toward Earth agrees with your answer from part (a).

Earth's radius is $R = 6.38 \times 10^3 \text{ km}$.

$$F = G \frac{M_E m}{r^2}$$

$$m g = G \frac{M_E m}{R^2}$$

$$a(R) = g = G \frac{M_E}{R^2}$$

$$a(60R) = G \frac{M_E}{(60 R)^2} = \frac{1}{3600} G \frac{M_E}{R^2} = \frac{1}{3600} g$$

$$a(60R) = \frac{1}{3600} g = \frac{1}{3600} (9.8 \frac{m}{s^2}) = 2.7 \times 10^{-3} \frac{m}{s^2} = 2.7 \frac{mm}{s^2}$$

$$a_c = \frac{v^2}{r}$$

$$v = \frac{C}{T} = \frac{2}{T} \frac{r}{T} = \frac{2}{T} \frac{(60 R)}{T} = \frac{2}{27.26 da} \frac{(60 m)}{24 hr} \frac{da}{3600 s}$$

$$v = 1,020 m/s$$

$$a_{c} = \frac{V^{2}}{r} = \frac{V^{2}}{60 R} = \frac{(1,020 m/s)^{2}}{60 x 6.38 x 10^{6} m} = 2.7 x 10^{-3} \frac{m}{s^{2}} = 2.7 \frac{m}{s^{2}}$$

5.49 What orbital radius should a weather satellite have if it is to have a period of 6.0 hours?

$$F_{c} = \mathbf{m} \frac{\mathbf{v}^{2}}{\mathbf{r}} = \mathbf{G} \frac{\mathbf{M}_{E}}{\mathbf{r}^{2}} \mathbf{m} = \mathbf{F}_{g}$$

$$\frac{\mathbf{v}^{2}}{\mathbf{r}} = \mathbf{G} \frac{\mathbf{M}_{E}}{\mathbf{r}^{2}}$$

$$\mathbf{v} = \frac{\mathbf{C}}{\mathbf{T}} = \frac{2}{\mathbf{T}} \mathbf{r}$$

$$\frac{\mathbf{v}^{2}}{\mathbf{r}} = \frac{4}{\mathbf{T}^{2}} \frac{\mathbf{r}}{\mathbf{r}^{2}} = \mathbf{G} \frac{\mathbf{M}_{E}}{\mathbf{r}^{2}}$$

$$\mathbf{r}_{3} = \frac{1}{4} \frac{\mathbf{r}}{2} \mathbf{G} \mathbf{M}_{g} \mathbf{T}_{2}$$

$$\mathbf{r}_{3} = \frac{1}{4} \frac{\mathbf{r}}{2} \mathbf{G} \mathbf{M}_{g} \mathbf{T}_{2}$$

$$\mathbf{r}_{3} = \frac{1}{4} \frac{\mathbf{r}}{2} (6.67 \times 10^{-11}) (5.98 \times 10^{24}) (6.0 \text{ h} \frac{3600 \text{ s}}{\text{ h}})^{2}$$

$$\mathbf{r}_{3} = 4.71 \times 10^{21}$$

$$\mathbf{r} = 1.68 \times 10^{7} \text{ m} = 1.68 \times 10^{4} \text{ km} = 16,800 \text{ km}$$

5.60 From the data in Table 5.1, calculate the acceleration of free fall on the surface of

a) Jupiter, b) Saturn, and c) our Moon. $F = m a = G \frac{M m}{r^2}$ $a = G \frac{M}{r^2}$ $a = 6.67 \times 10^{-11} \frac{1,900 \times 10^{24}}{(71,400,000)^2} = 24.9 \frac{m}{s^2}$ a) $a = 6.67 \times 10^{-11} \frac{561 \times 10^{24}}{(71,400,000)^2} = 10.4 \text{ m}$

b)
$$a = 6.67 \times 10^{-11} \frac{501 \times 10}{(60,000,000)^2} = 10.4 \frac{\text{m}}{\text{s}^2}$$

c)
$$a = 6.67 \times 10^{-11} \frac{0.0736 \times 10^{24}}{(1,740,000)^2} = 1.62 \frac{m}{s^2}$$