

13.5 A goose-down sleeping bag has a surface area of 2.25 m^2 and is filled iwth a layer of down 5 cm thick. What is the heat-transfer rate (H_{cd}) through it from a person with skin temperature of 35°C to the outside air at -5°C . How does this rate compare with the body's minimal metabolism rate, about 100 W?

From Table 13,1, p 457, we find the thermal conductivity of goose down to be

$$K = 0.023 \text{ W /m C}^\circ$$

We can now apply the conduction heat-transfer equation, Eq 13.1,

$$H_{cd} = [KA(T_2 - T_1)] / L$$

From the problem, we know the other pieces of this equation,

$$T = T_2 - T_1 = 35^\circ\text{C} - (-5^\circ\text{C}) = 40 \text{ C}^\circ$$

$$A = 2.25 \text{ m}^2$$

$$L = 5 \text{ cm} = 0.05 \text{ m}$$

$$H_{cd} = [KA(T_2 - T_1)] / L$$

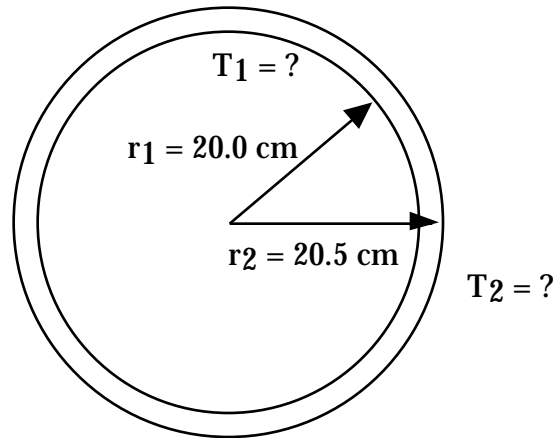
$$H_{cd} = [(0.023\text{W/m C}^\circ)(2.25 \text{ m}^2)(40 \text{ C}^\circ)] / 0.05 \text{ m} =$$

$$H_{cd} = 41 \text{ W}$$

This is about half of the body's normal metabolism so this sleeping bag should keep our camper a "happy camper" (or, at least, a warm camper).

13.17 A lamp bulb inside an ornamental spherical glass shell 20 cm in radius and 0.50 cm thick radiates 100 W of thermal power. What is the difference in temperature between the inner and outer surfaces of the glass?

(Hmmm, . . . go ahead and use $e = 1.00$, even in this case).



To find the temperature, we can use the Radiation equation, Eq 13.4,

$$R_{\text{emit}} = e A T^4$$

We will use $e = 1.0$, even for this. σ is the Stefan-Boltzman constant,

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

The surface area of a sphere is

$$A = (4/3) \pi r^2$$

For the inner surface,

$$A_{\text{inner}} = (4/3) \pi (0.200 \text{ m})^2 = 0.1676 \text{ m}^2$$

For the outer surface,

$$A_{\text{out}} = (4/3) \pi (0.205 \text{ m})^2 = 0.1760 \text{ m}^2$$

The temperatures T must be absolute temperatures, measured on the Kelvin scale.

Now we are ready to solve for the temperatures

$$T^4 = R_{\text{emit}} / e A$$

$$T_{\text{inner}}^4 = 100 \text{ W} / [(1.0)(5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4)(0.1676 \text{ m}^2)] = 1.0523 \times 10^{10} \text{ K}^4$$

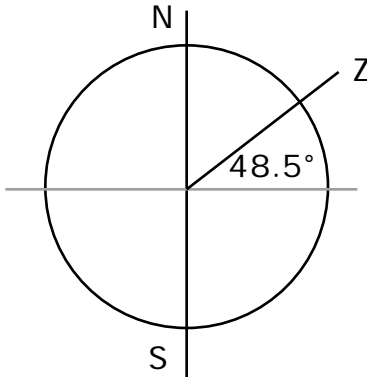
$$T_{\text{inner}} = 320.3 \text{ K}$$

$$T_{\text{outer}}^4 = 100 \text{ W} / [(1.0)(5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4)(0.1760 \text{ m}^2)] = 1.0078 \times 10^{10} \text{ K}^4$$

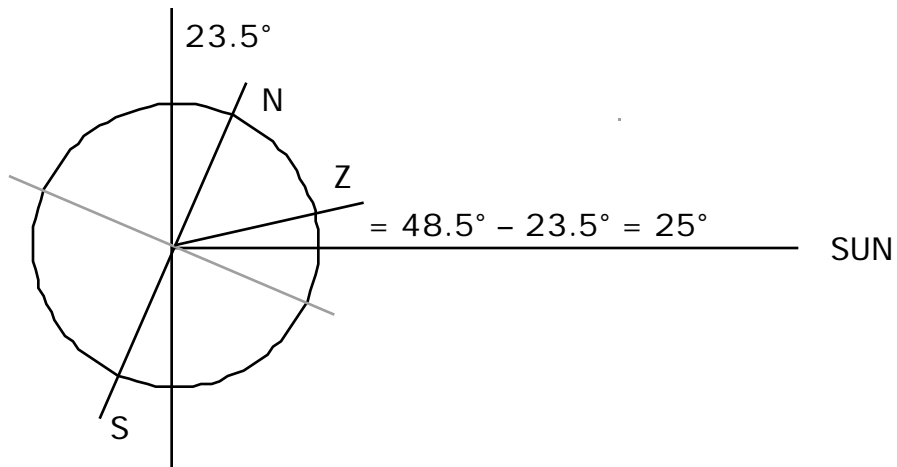
$$T_{\text{outer}} = 316.8 \text{ K}$$

Our use of $e = 1.0$ makes our actual numbers rather “iffy”. But the point of this problem was to show that the inner surface will be at a higher temperature than the outer surface. That is certainly true for our calculation. Changing the value of the emissivity e may change our actual numbers but we will still find $T_{\text{inner}} > T_{\text{outer}}$.

13. 21 Victoria, Canada, is located at 48.5° north latitude. How high above the horizon is the Sun on the first day of summer and the first day of winter?

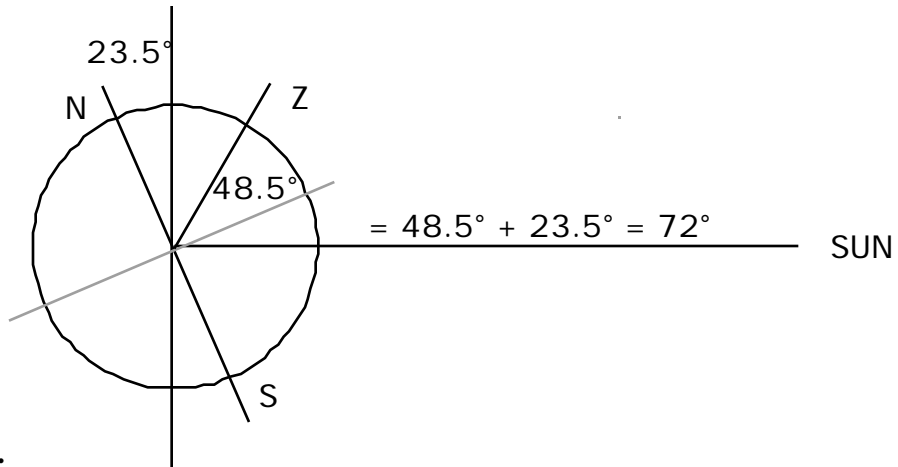


N and S mark the poles, the axis of rotation. Z is the zenith, the point directly overhead, 48.5° from the equator (48.5° north latitude).



Summer:

is the angle from directly overhead (the zenith) to the Sun. Therefore, the angle between the horizon and the Sun will be $90^\circ - 25^\circ = 65^\circ$. The Sun will be 65° above the horizon at the summer solstice.

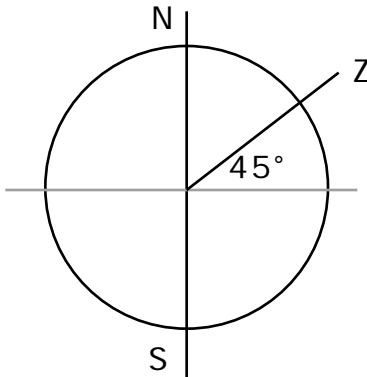


Winter:

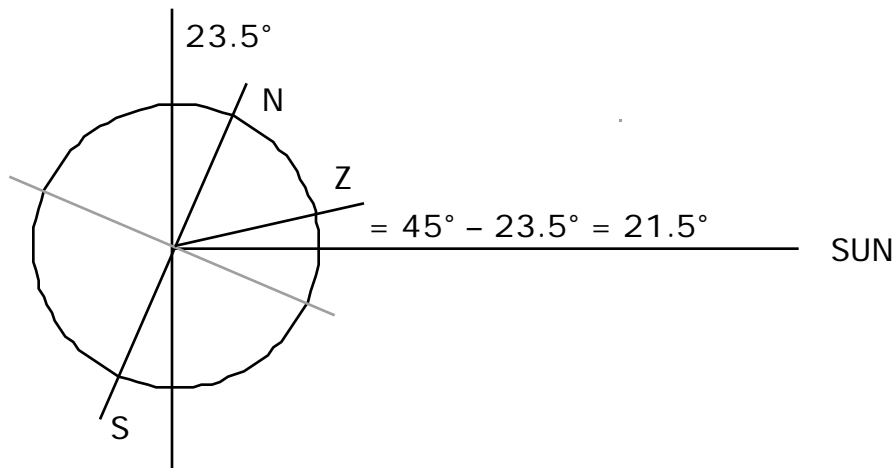
is again the angle between directly overhead (the zenith) and the Sun. Therefore, the Sun is $90^\circ - 72^\circ = 18^\circ$ above the horizon at the winter solstice.

13.24 A solar home is to be built in Minneapolis, 45° north latitude. What is the elevation of the Sun at midday on the first day of summer and on the first day of winter? An overhanging eave 85 cm wide is planned. How far below this eave will sunlight fall on the first day of summer and the first day of winter.

The first part of this questions requires exactly the same reasoning, using exactly the same kind of diagrams, as question 13.21.

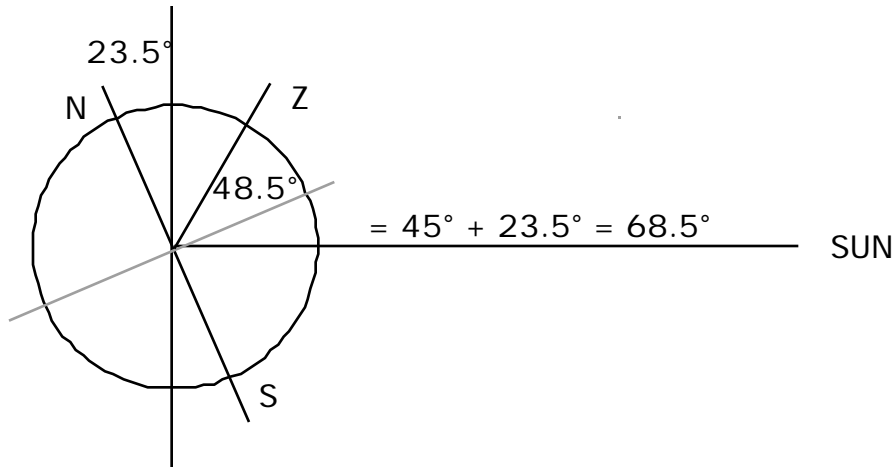


N and S mark the poles, the axis of rotation. Z is the zenith, the point directly overhead, 45° from the equator (45° north latitude).



Summer:

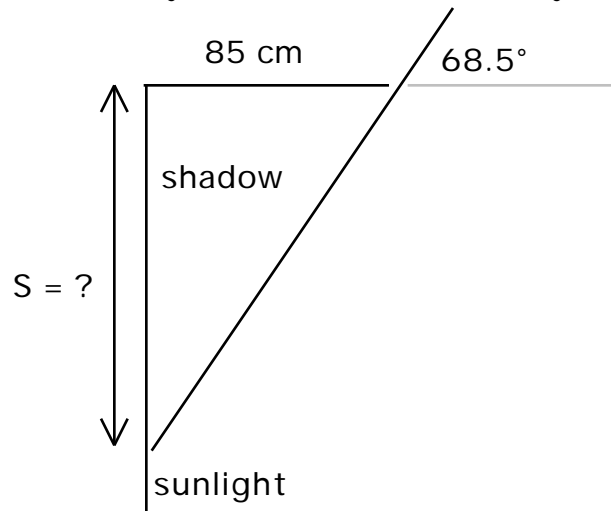
If the Sun is 21.5° from the zenith (the position directly overhead) then it must be $90^\circ - 21.5^\circ = 68.5^\circ$ above the horizon.



Winter:

If the Sun is 68.5° from the zenith, then it must be $90^\circ - 68.5^\circ = 21.5^\circ$ above the horizon.

Now we can look at the shadow cast by the eave on these two days:



Summer:

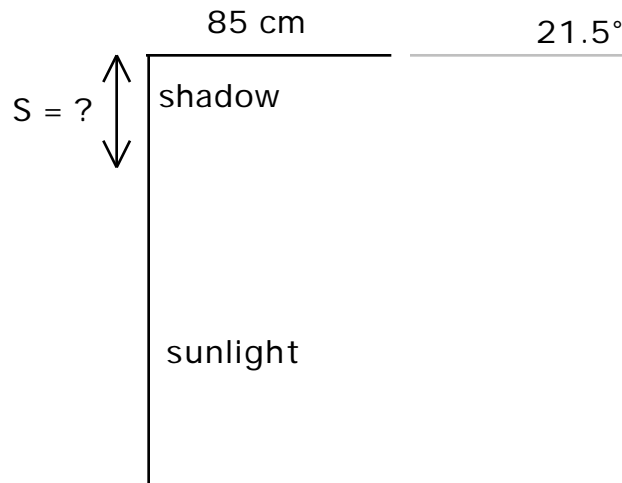
The length of the shadow, S in the diagram, is the opposite side of a 68.5° right triangle (85 cm is the adjacent side).

$$\tan = \text{opp}/\text{adj}$$

$$\tan 68.5^\circ = S / 85 \text{ cm}$$

$$S = (85 \text{ cm})(\tan 68.5^\circ)$$

$$S = (85 \text{ cm})(2.54) = 216 \text{ cm} = 2.16 \text{ m}$$



Winter:

The length of the shadow, S in the diagram, is the opposite side of a 21.5° right triangle (85 cm is the adjacent side).

$$\tan = \text{opp/adj}$$

$$\tan 21.5^\circ = S / 85 \text{ cm}$$

$$S = (85 \text{ cm})(\tan 21.5^\circ)$$

$$S = (85 \text{ cm})(0.393) = 33.5 \text{ cm}$$

13.26 Calculate the R value of an insulated wall like that in Figure 13.30 if 2 x 6's are used for wall studs instead of 2 x 4's. The larger board allow 5.5 in of fiberglass insulation instead of 3.5 in.

By what percentage does this increase in thickness change the R value?

How does the increased thickness change the heat loss calculated in Example 13.14?

The R-values for the components of the wall in Figure 13.30 are given in Table 13.4, on page 481. By increasing the fiberglass insulation from 3.5 inches to 5.5 inches, the R-value for this insulation increases from 10.90 to 18.80, according to Table 13.3, on page 480. This increases the total R-value from 14.33 to **22.23**.

This is an increase of $7.9/14.33 = 0.55 = 55\%$ for the R-value of the wall.

In terms of R-values, the conduction heat transfer rate is given by equation 13.9,

$$H_{cd} = A(T_2 - T_1) / R = (1/R) [A (T_2 - T_1)]$$

In Example 13.14, we found the conduction heat transfer rate to be

$$H_{cd}(\text{un}) = 3.8 \text{ kW for the } \underline{\text{uninsulated}} \text{ wall,}$$

$H_{cd}(3.5) = 1.2 \text{ kW}$ for the insulated wall with 2x4's and 3.5 in of insulation

Now, with 2x6's and 5.5 in of insulation,

$$H_{cd}(5.5) = (1/22.23)[(960)(60)] = 2591 \text{ Btu/h} = 0.76 \text{ kW}$$

This is a reduction of heat flow by

$$H = 1.18 \text{ kW} - 0.76 \text{ kW} = 0.42 \text{ kW}$$

$$H / H = 0.42 \text{ kW} / 1.18 \text{ kW} = 0.36 = 36\%$$

13.30 A roasted turkey cools from 85°C to 80°C in 10 min when sitting in a 25°C room. How long does it require to cool from 85°C to 55°C?

This requires an application of Newton's Law of Cooling, equation 13.3,

$$T(t) = T_{sur} + T e^{-t/\tau}$$

T is the initial temperature difference of the turkey and its surroundings;

$$T = 85^\circ\text{C} - 25^\circ\text{C} = 60 \text{ C}^\circ$$

Knowing that it cools from 85°C to 80°C allows us to solve for the "time constant" in this equation;

$$T(t) = T_{sur} + T e^{-t/\tau}$$

$$T(10 \text{ min}) = 80^\circ\text{C} = 25^\circ\text{C} + (60 \text{ C}^\circ) e^{-(10 \text{ min})/\tau}$$

$$55^\circ\text{C} = (60 \text{ C}^\circ) e^{-(10 \text{ min})/\tau}$$

$$55^\circ\text{C}/60^\circ\text{C} = e^{-(10 \text{ min})/\tau}$$

$$55/60 = e^{-(10 \text{ min})/\tau}$$

$$0.9167 = e^{-(10 \text{ min})/\tau}$$

$$e^{-(10 \text{ min})/\tau} = 0.9167$$

$$\ln[e^{-(10 \text{ min})/\tau}] = \ln[0.9167]$$

$$-(10 \text{ min})/\tau = -0.0870$$

$$10 \text{ min}/\tau = 0.0870$$

$$= (10 \text{ min})/0.0870$$

$$= 115 \text{ min}$$

Now we know the time constant and we can use Newton's Law of Cooling to go back and solve for t, the time, when T, the temperature, is 55°C.

$$T(t) = T_{sur} + T e^{-t/\tau}$$

$$T(t) = 55^\circ\text{C} = 25^\circ\text{C} + (60 \text{ C}^\circ) e^{-t/(115 \text{ min})}$$

$$55^{\circ}\text{C} = 25^{\circ}\text{C} + (60^{\circ}\text{C}) e^{-t/(115 \text{ min})}$$

$$30^{\circ}\text{C} = (60^{\circ}\text{C}) e^{-t/(115 \text{ min})}$$

$$30^{\circ}\text{C}/60^{\circ}\text{C} = e^{-t/(115 \text{ min})}$$

$$30/60 = e^{-t/(115 \text{ min})}$$

$$0.50 = e^{-t/(115 \text{ min})}$$

$$\ln[0.50] = \ln[e^{-t/(115 \text{ min})}]$$

$$-0.693 = -t/(115 \text{ min})$$

$$0.693 = t/(115 \text{ min})$$

$$t = (0.693)(115 \text{ min})$$

$$t = 78 \text{ min}$$