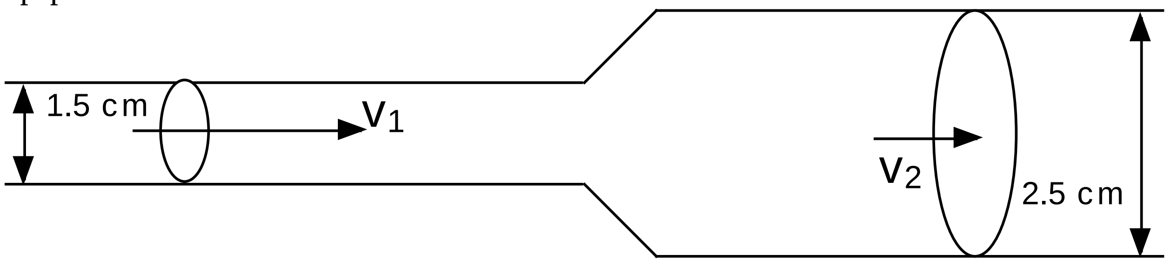
11.1 Water flows through a pipe at the rate of 0.35 m^3/ min. What is its speed where the pipe has a diameter of 1.5 cm? Where the diameter is 2.5 cm?



$$\frac{\Delta m}{\Delta t} = 3\,\frac{m^3}{min} = 3\,\frac{m^3}{min}\left[\frac{min}{60\ sec}\right] = 0.05\,\frac{m^3}{sec}$$

$\Delta m / \Delta t = \rho\ A\ v$ = constant

$\rho = 1000\ kg/m^3$

$A_1 = \pi r_1^2 = \pi (0.75\ cm)^2 = 1.77\ cm^2$

$A_2 = \pi r_2^2 = \pi (1.25\ cm)^2 = 4.91\ cm^2$

$v = [\Delta m / \Delta t\,][1/\rho\ A]$

$v_1 = [\Delta m / \Delta t\,][1/\rho\ A1] = [0.05\ m^3/s][1/(1000\ kg/m^3)(\ 1.77\ cm^2)]$

Be <u>very</u> careful of the units!

$A_1 = 1.77\ cm^2 = 1.77\ cm^2\ [1\ m\ /\ 100\ cm]^2 = 0.000177\ m^2 = 1.77 \times 10^{-4}\ m^2$

$v_1 = [\Delta m / \Delta t\,][1/\rho\ A1] = [0.05\ m^3/s][1/(1000\ kg/m^3)(\ 1.77 \times 10^{-4}\ m^2)]$

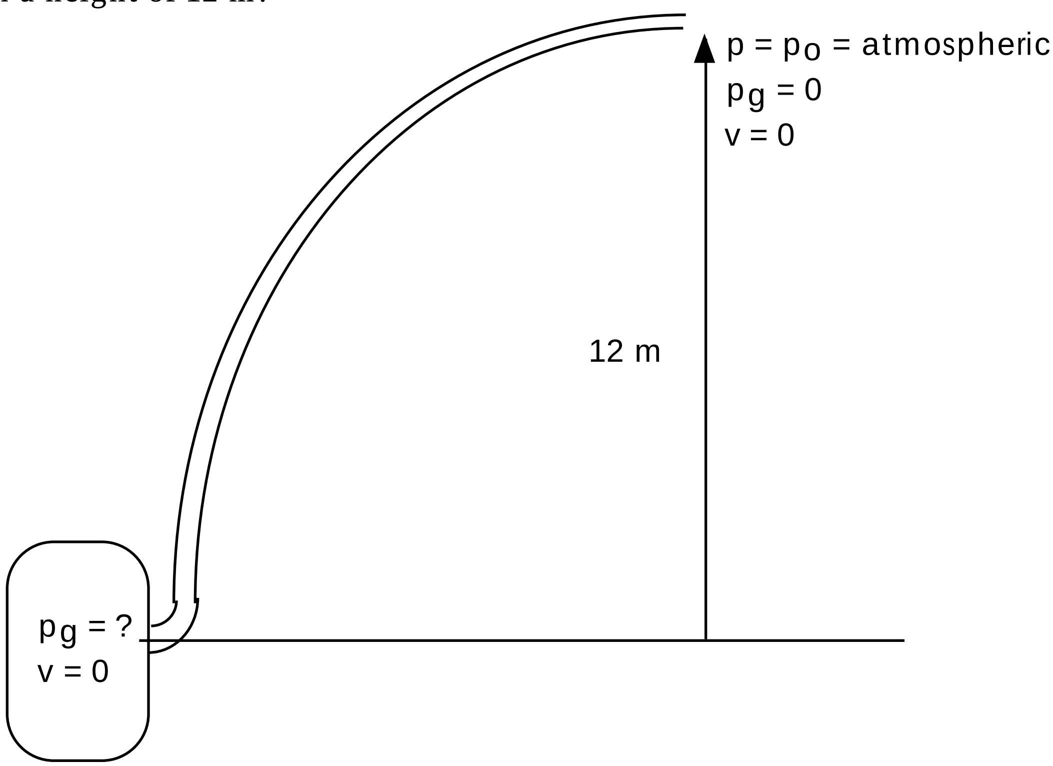
$v_1 = 0.28\ m/s = 28\ cm/s$

$A_2 = 4.91\ cm^2 = 4.91\ cm^2\ [1\ m\ /\ 100\ cm]^2 = 0.000\,491\ m^2 = 4.91 \times 10^{-4}\ m^2$

$v_2 = [\Delta m / \Delta t\,][1/\rho\ A2] = [0.05\ m^3/s][1/(1000\ kg/m^3)(\ 4.91 \times 10^{-4}\ m^2)]$

$v_2 = 0.102\ m/s = 10.2\ cm/s$

11.6 What must be the gauge pressure at a fire hydrant if water from a fire hose is to reach a height of 12 m?



We can immediately apply Bernoulii's Equation,

$\frac{1}{2}\rho v^2 + \rho g h + p = \text{constant}$

or

$\frac{1}{2}\rho v_1^2 + \rho g h_1 + p_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + p_2$

In our case, the hydrant is position **1** and the top of the stream of water is position **2**,

$v_1 = 0$, $h_1 = 0$, $p_1 = p_{1g} = ?$

$v_2 = 0$, $h_2 = 12$ m, $p_2 = p_{2g} = 0$ (the guage pressure at the top of the stream is zero).

$\frac{1}{2}\rho (0)^2 + \rho g (0) + p_{1g} = \frac{1}{2}\rho (0)^2 + \rho g (12 \text{ m}) + (0)$

$p_{1g} = \rho g (12 \text{ m})$

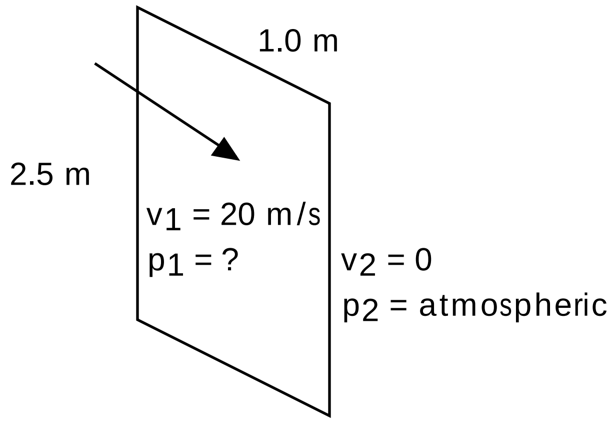
$p_{1g} = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(12 \text{ m})$

$p_{1g} = 117{,}600 \text{ N/m}^3 = 117{,}600 \text{ Pa} = 117.6 \text{ kPa}$

$\mathbf{p_{1g} = 117.6\ kPa}$

$p_{1g} = 117.6 \text{ kPa} \left[\frac{1 \text{ atm}}{101.3 \text{ kPa}}\right] = 1.16 \text{ atm}$

11.7 Air (ρ = 1.3 kg/m^3) blows past a 1.0 m x 2.5 m window with a speed of 20 m/s. What is the difference in pressure on the two sides of the window? What is the net force on the window?



We can (again) immediately apply Bernoulii's Equation,

$^1/_2 \rho v^2 + \rho g h + p = \text{constant}$

or

$^1/_2 \rho v_1^2 + \rho g h_1 + p_1 = {}^1/_2 \rho v_2^2 + \rho g h_2 + p_2$

In our case, position **1** is <u>outside</u> the window and position **2** is <u>inside</u> the window. The height is the same so $h_1 = h_2 = h$,

$^1/_2 \rho v_1^2 + \rho g h_1 + p_1 = {}^1/_2 \rho v_2^2 + \rho g h_2 + p_2$

$^1/_2 \rho (20 \text{ m/s})^2 + \rho g h + p_1 = {}^1/_2 \rho (0)^2 + \rho g h + p_2$

$^1/_2 \rho (20 \text{ m/s})^2 + [\rho g h] + p_1 = {}^1/_2 \rho (0)^2 + [\rho g h] + p_2$

$^1/_2 \rho (20 \text{ m/s})^2 = p_2 - p_1$

$p_2 - p_1 = {}^1/_2 \rho (20 \text{ m/s})^2 = {}^1/_2 (1.3 \text{ kg/m}^3)(20 \text{ m/s})^2 = 260 \text{ N/m}^2 = 260 \text{ Pa}$

Δp = 260 Pa

$F = \Delta p\, A = (260 \text{ N/m}^2)(1.0 \text{ m x } 2.5 \text{ m}) = 650 \text{ N}$

F = 650 N

11.11 Below are data for the cross-sectional area of open pipes and the pressure head behind each. (The "pressure head" is the gauge pressure measured in meters of water). Find the volume flow rate through each pipe.

pipe	cross-section area (cm^2)	pressure head (m)
a	1.0	2.5
b	1.2	2.5
c	1.5	2.5
d	1.8	3.0
e	3.0	4.0
f	3.0	5.0
g	5.0	6.0

This will be a good problem to solve once algebraicly and then use a spreadsheet to solve for each particular set of numbers.

We begin, again, with Bernoulii's Equation,

$^1/_2 \rho v^2 + \rho g h + p = \text{constant}$

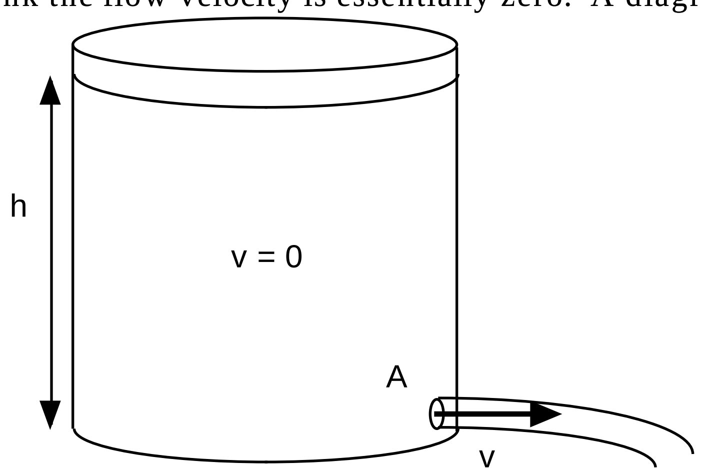
or

$^1/_2 \rho v_1^2 + \rho g h_1 + p_1 = {^1/_2} \rho v_2^2 + \rho g h_2 + p_2$

We are not concerned with any difference in height, so we may set $h_1 = h_2 = h$. Then our equation becomes

$^1/_2 \rho v_1^2 + p_1 = {^1/_2} \rho v_2^2 + p_2$

We can think of the flow rate due to a pressure difference as the flow out of a large tank. Inside the tank the flow velocity is essentially zero. A diagram may help—



$$^1/_2 \rho v_1^2 + p_1 = {^1/_2} \rho v_2^2 + p_2$$

$$^1/_2 \rho (0)^2 + p_1 = {^1/_2} \rho v_2^2 + p_2$$

$$p_1 - p_2 = {^1/_2} \rho v_2^2$$

$$^1/_2 \rho v_2^2 = p_1 - p_2$$

$$ {}^{1}/{}_{2}\, \rho\, v_2^{2} = \Delta p $$

However, this difference in pressure is often refered to as a “pressure head” and is then stated in terms of the height of a column of water that would provide this difference in pressure; that is

$$ \Delta p = \rho\, g\, h $$

$$ {}^{1}/{}_{2}\, \rho\, v_2^{2} = \Delta p $$

$$ {}^{1}/{}_{2}\, \rho\, v_2^{2} = \rho\, g\, h $$

$$ {}^{1}/{}_{2}\, v_2^{2} = g\, h $$

$$ v_2^{2} = 2\, g\, h $$

$$ v = \sqrt{2\, g\, h} $$

(This is also known as Torecelli’s Theorem).

Now that we know the velocity, the volume flow rate is just the velocity multiplied by the cross-sectional area,

$$ \text{volume flow rate} = A\, v $$

$$ \text{volume flow rate} = A\, \sqrt{2\, g\, h} $$

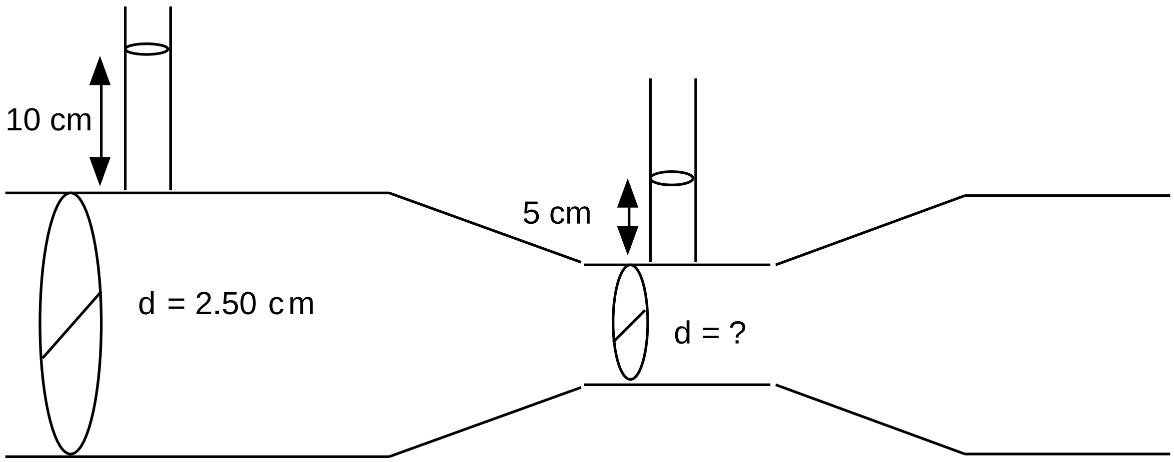
Now we are ready to create a spreadsheet to do the actual calculations. However, a word of caution is appropriate here. <u>Be careful with the units!</u>

	A	B	C	D	E
1		c-s area	c-s area	press head	**vol flow rate**
2		cm*cm	m*m	m	**m^3/s**
3	a	1.00	0.00010	2.50	**0.00070**
4	b	1.20	0.00012	2.50	**0.00084**
5	c	1.50	0.00015	2.50	**0.00105**
6	d	1.80	0.00018	3.00	**0.00138**
7	e	3.00	0.00030	4.00	**0.00266**
8	f	3.00	0.00030	5.00	**0.00297**
9	g	5.00	0.00050	6.00	**0.00542**
10					
11					
12					

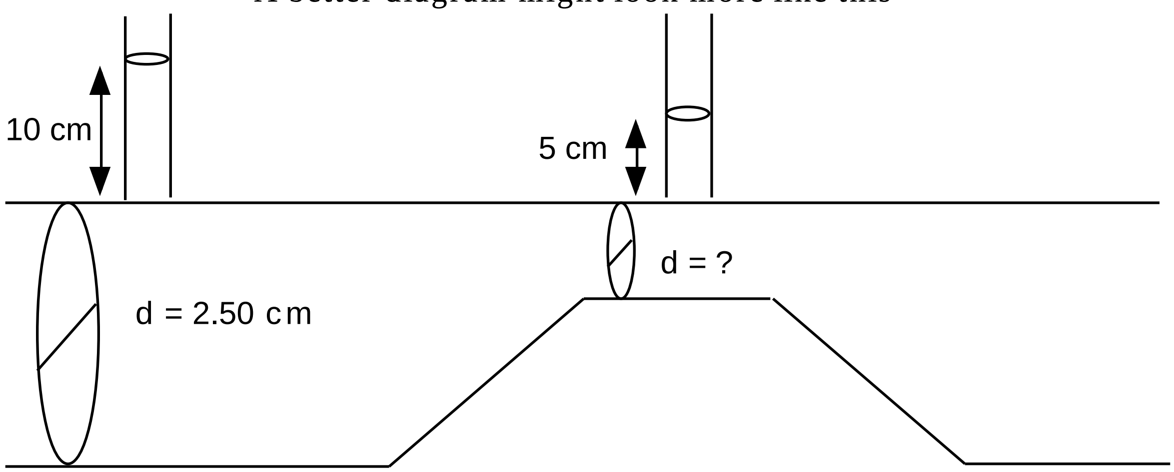
As an example, the formula for pipe a, which appears in cell E3, is

=C3*SQRT(2*9.8*D3)

11.20 The inside diameter of the larger part of the pipe in Figure 11.22 is 2.50 cm. Water flows through the pipe at a rate of 1.80 l/s. What is the inside diameter of the constriction?



A better diagram might look more like this—



We begin, again, with Bernoulii's Equation,

$^1/_2\,\rho\, v^2 + \rho\, g\, h + p = \text{constant}$

or

$^1/_2\,\rho\, v_1{}^2 + \rho\, g\, h_1 + p_1 = {}^1/_2\,\rho\, v_2{}^2 + \rho\, g\, h_2 + p_2$

We are not concerned with any difference in height, so we may set $h_1 = h_2 = h$. Then our equation becomes

$^1/_2\,\rho\, v_1{}^2 + p_1 = {}^1/_2\,\rho\, v_2{}^2 + p_2$

We <u>know</u> the volume flow rate of 0.180 l/s. In the larger part of the pipe, we know the cross-section area A_1,

$A_1 = \pi\, r_1 2 = \pi\,(1.25\text{ cm})^2 = \pi\,(0.0125\text{ m})^2 = 4.91 \times 10^{-4}\ \text{m}^2$

volume flow rate = $Av = A_1\, v_1$

v_1 = [volume flow rate]/A_1 = $[0.180 \times 10^{-3}\ \text{m}^3/\text{s}]/[4.91 \times 10^{-4}\ \text{m}^2]$

$v_1 = 0.367\ m/s$

where we have made use of

$$1\ \text{liter} = 1\ l = 10^{-3}\ m^3$$

$$^1/_2\, \rho\, v_1^2 + p_1 = {}^1/_2\, \rho\, v_2^2 + p_2$$

$$^1/_2\, \rho\, (0.367\ m/s)^2 + p_1 - p_2 = {}^1/_2\, \rho\, v_2^2$$

$$^1/_2\, \rho\, v_2^2 = {}^1/_2\, \rho\, (0.367\ m/s)^2 + p_1 - p_2$$

$$v_2^2 = (0.367\ m/s)^2 + 2\,(p_1 - p_2)/\rho$$

We know the pressures p_1 and p_2 in terms of the "pressure head", the height of the column of <u>water</u> that is equivalent to these pressures.

$p = \rho\, g\, h$

$p_1 = \rho\, g\, h_1 = (1000\ {}^{kg}/_{m^3})\ (9.8\ m/s^2)\ (\ 0.10\ m)$

$p_2 = \rho\, g\, h_2 = (1000\ {}^{kg}/_{m^3})\ (9.8\ m/s^2)\ (\ 0.05\ m)$

$p_2 - p_1 = \rho\, g\, h_2 - \rho\, g\, h_1$

$2\,(p_1 - p_2)/\rho = 2\,(\rho\, g\, h_2 - \rho\, g\, h_1)/\rho = g\,(h_2 - h_1) = (9.8\ m/s^2)(0.05\ m) = 0.49\ m^2/s^2$

$$v_2^2 = (0.367\ m/s)^2 + 2\,(p_1 - p_2)/\rho$$

$$v_2^2 = (0.367\ m/s)^2 + 0.49\ m^2/s^2$$

$$v_2^2 = 0.13\ m^2/s^2 + 0.49\ m^2/s^2$$

$$v_2^2 = 0.13\ m^2/s^2 + 0.49\ m^2/s^2$$

$$v_2^2 = 0.62\ m^2/s^2$$

$$v_2 = 0.79\ m/s$$

This is the <u>velocity</u> through the constriction. What area (or <u>diameter</u>) will provide this velocity?

$$\text{volume flow rate} = A v = A_2\, v_2$$

$$A_2 = [\text{volume flow rate}]/v_2 =$$

$$A_2 = [0.180 \times 10^{-3}\ m^3/s]/[0.79\ m/s] =$$

$$A_2 = 2.28 \times 10^{-4}\ m^2 = \pi\, r^2$$

$$r^2 = 7.25 \times 10^{-5}\ m^2$$

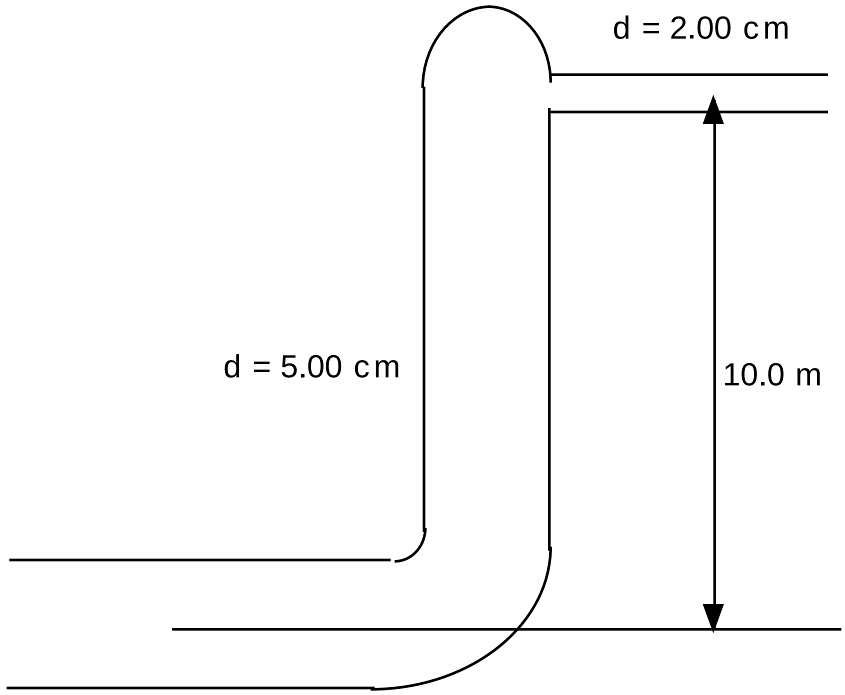
$$r = 8.5 \times 10^{-3}\ m$$

$$d = 2r = 1.70 \times 10^{-2}\ m$$

$$d = 1.7\ cm$$

(finally!)

11.22 The dimensions given in Figure 11.24 are all inside diameters. When the faucet valve is wide open, it does not constrict the smaller pipe. If water enters the larger pipe under a pressure of 3800 torr, what volume emerges from the faucet per second. Compare this exact result with the restult obtained by disregarding the speed of the water in the larger pipe. Assume the pressure in the open faucet is 760 torr (1 atm).



In the large pipe, at position 1,

$p_1 = 3800$ torr = 3800 torr [133 Pa / 1 torr] = 505,000 Pa = 505 kPa

$h_1 = 0$

$A_1 = \pi r_1^2 = \pi (0.0025\ m)^2 = 1.96 \times 10^{-5}\ m^2$

In the small pipe, at position 2,

$p_2 = 760$ torr = 760 torr [133 Pa / 1 torr] = 101,000 Pa = 101 kPa

$h_2 = 10$ m

$A_2 = \pi r_2^2 = \pi (0.001\ m)^2 = 3.14 \times 10^{-6}\ m^2$

As always, it would seem, we shall apply Bernoulii's Equation,

$^1/_2\, \rho\, v^2 + \rho\, g\, h + p = \text{constant}$

or

$^1/_2\, \rho\, v_1^2 + \rho\, g\, h_1 + p_1 = {^1/_2}\, \rho\, v_2^2 + \rho\, g\, h_2 + p_2$

VFR = volume flow rate = $A_1\, v_1 = A_2\, v_2$

$v_1 = VFR / A_1; \quad v_1^2 = [VFR / A_1]^2$

$v_2 = VFR / A_2; \quad v_2^2 = [VFR / A_2]^2$

Now we know everything in Bernoulii's Equation, except the volume flow rate

(VFR) so we can (finally) solve for that!

$^1/_2 \rho\, v_1^2 + \rho\, g\, h_1 + p_1 = {}^1/_2 \rho\, v_2^2 + \rho\, g\, h_2 + p_2$

$^1/_2 \rho\ [VFR / A_1]^2 + \rho\, g\, h_1 + p_1 = {}^1/_2 \rho\ [VFR / A_2]^2 + \rho\, g\, h_2 + p_2$

$[VFR / A_1]^2 + 2\, g\, h_1 + 2\, p_1/\rho = [VFR / A_2]^2 + 2\, g\, h_2 + 2\, p_2/\rho$

$[VFR / A_1]^2 - [VFR / A_2]^2 = 2\, g\, (h_2 - h_1) + 2\, (p_2 - p_1)/\rho$

$VFR^2\, [\, (1 / A_1)^2 - (1 / A_2)^2\,] = 2\, g\, (h_2 - h_1) + 2\, (p_2 - p_1)/\rho$

$VFR^2\, [(A_2^2 - A_1^2) / (A_1^2\, A_2^2)] = 2\, g\, (h_2 - h_1) + 2\, (p_2 - p_1)/\rho$

$VFR^2 = [\, 2\, g\, (h_2 - h_1) + 2\, (p_2 - p_1)/\rho]\, [(A_1^2\, A_2^2) / (A_2^2 - A_1^2)]$

Now (finally!) we can plug in numbers,

$VFR^2 = [2\, (9.8\ m / s^2)\, (10\ m - 0) + 2\, (-\, 404{,}000\ ^N/_{m^2}) / (1000\ ^{kg}/_{m^3})]$ x

x $[((\, 3.14 \times 10^{-6}\ m^2)^2\, (1.96 \times 10^{-5}\ m^2)^2) / ((\, 3.14 \times 10^{-6}\ m^2)^2 - (1.96 \times 10^{-5}\ m^2))^2)]$

(messy, eh? Sure, but fairly straightforward, nonetheless).

$VFR^2 = [196\ m^2/s^2 - 808\ m^2/s]\, [(3.79 \times 10^{-21}\ m^8) / (9.86 \times 10^{-12} - 384 \times 10^{-12})\ m^4]$

$VFR^2 = [196\ m^2/s^2 - 808\ m^2/s]\, [(3.79 \times 10^{-21}\ m^8) / (-\, 374 \times 10^{-12}\ m^4)]$

$VFR^2 = [\, -\, 612\ m^2/s^2\,]\, [\, -\, 1.01 \times 10^{-11}\ m^4\,]$

$VFR^2 = 6.202 \times 10^{-9}\ m^6/s^2$

$VFR\ = 7.88 \times 10^{-5}\ m^3/s$

$VFR\ = 7.88 \times 10^{-5}\ m^3/s\, [\ ^{1000\ l}/_{s}\ ] = 0.0788\ l/s$ (liters per second)

Regard the larger pipe as a large tank with $v_1 = 0$. Then Bernoulii's Equation is

$^1/_2 \rho\, v_1^2 + \rho\, g\, h_1 + p_1 = {}^1/_2 \rho\, v_2^2 + \rho\, g\, h_2 + p_2$

$0 + \rho\, g\, h_1 + p_1 = {}^1/_2 \rho\, v_2^2 + \rho\, g\, h_2 + p_2$

$^1/_2 \rho\, v_2^2 + \rho\, g\, h_2 + p_2 = \rho\, g\, h_1 + p_1$

$^1/_2 \rho\, v_2^2 = \rho\, g\, (h_1 - h_2) + (p_1 - p_2)$

$v_2^2 = 2\, g\, (h_1 - h_2) + 2\, (p_1 - p_2)/\rho$

$v_2^2 = 2(9.8\ m / s^2)\, (0 - 10\ m) + 2\, (505\ kPa - 101\ kPa) / (1000\ kg / m^3)$

$v_2^2 = -\, 196\ m^2/s^2 + 808\ m^2/s^2$

$v_2^2 = 612\ m^2/s^2$

$v_2 = 24.7\ m / s$

This number seems large, but believable. Now, what is the volume flow rate?

VFR = volume flow rate = $A_2\, v_2 = (3.14 \times 10^{-6}\ m^2)(24.7\ m / s) =$

$VFR = 7.77 \times 10^{-5}\ m^3/s\, [\, 1000\ l / m^3\,] = 0.077\ l / s$ (liters per second)

And that seems like a reasonable number. The answers in the two cases are very similar so treating the larger pipe as a large tank is a very reasonable approximation.

Be sure to check on units!

11.37 The heart of a person sitting quietly pumps blood at the rate of $4 \times 10^3\ cm^3/min$ through an aorta of cross section $0.8\ cm^2$.
a) What is the average blood speed in the aorta?
The blood then spreads out into a network of about 5×10^9 (!) tiny capillaries whose average radius is about 8×10^{-4} cm.
b) What is the speed of the blood through these capillaries?

VFR = volume flow rate = A v
$v = {}^{VFR}/_{A} = [4 \times 10^3\ cm^3/min] \ /\ [0.8\ cm^2] = 5{,}000\ cm/min = 50\ m/min$
$v = {}^{50\ m}/_{min}\ [{}^{min}/_{60\ s}] = 0.83\ m/s$

What is the total area of the capillaries?
$A_{each} = \pi r^2 = (3.14)(8 \times 10^{-4}\ cm)^2 = (3.14)(8 \times 10^{-6}\ m)^2 = 2.01 \times 10^{-10}\ m^2$
While each capillary has a very small cross section, there are five billion of them!
$A_{total} = n\ A_{each} = (5 \times 10^9)(2.01 \times 10^{-10}\ m^2) = 1.00\ m^2 = 1.00 \times 10^4\ cm^2$

VFR = volume flow rate = A v
$v = {}^{VFR}/_{A} = [4 \times 10^3\ cm^3/min] \ /\ [1.00 \times 10^4\ cm^2] = 4 \times 10^{-1}\ cm/min$
$v = {}^{0.4\ cm}/_{min}\ [{}^{min}/_{60\ s}] = 6.67 \times 10^{-3}\ {}^{cm}/_{s}$
As you should expect, that velocity is very slow because the total cross section area is very large.