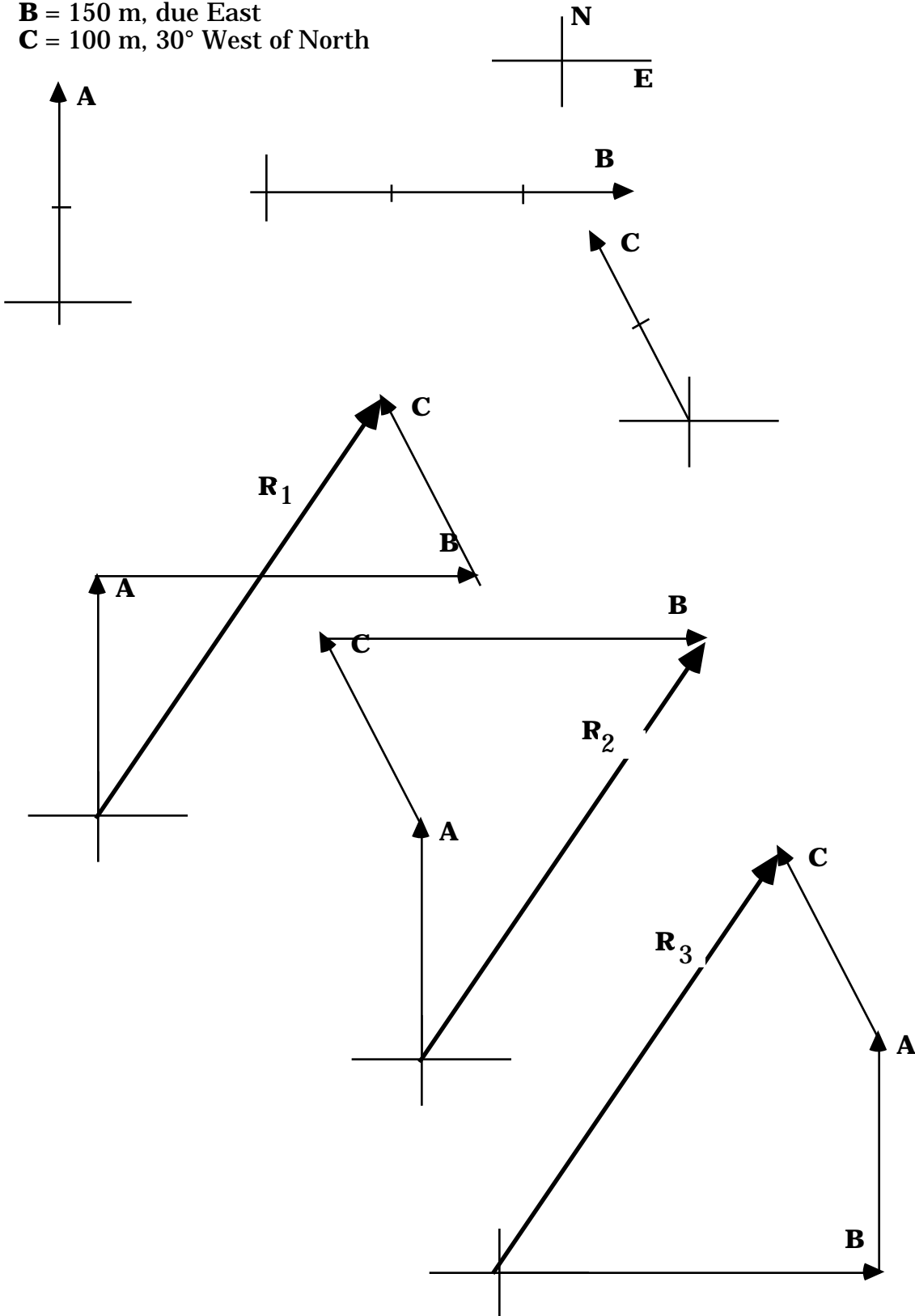
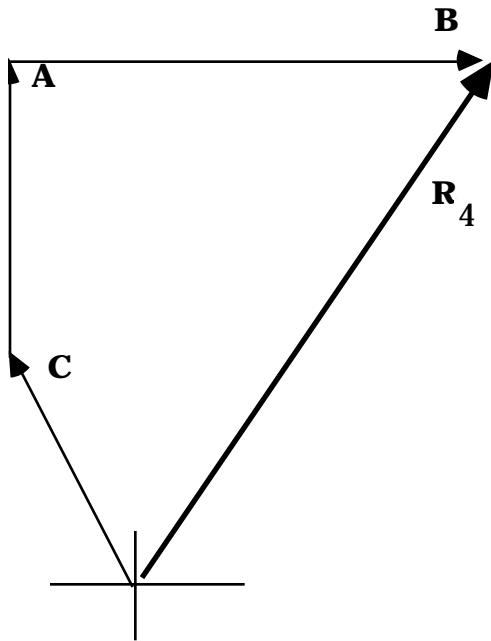


**PHY 1150**  
**Doug Davis**  
**Chapter 3, Vectors and Motion in Two Dimensions**  
**Ch 3: 8, 15, 23, 35, 41, 43, 54, 67**

- 3.8** **A** = 100 m, due North  
**B** = 150 m, due East  
**C** = 100 m, 30° West of North





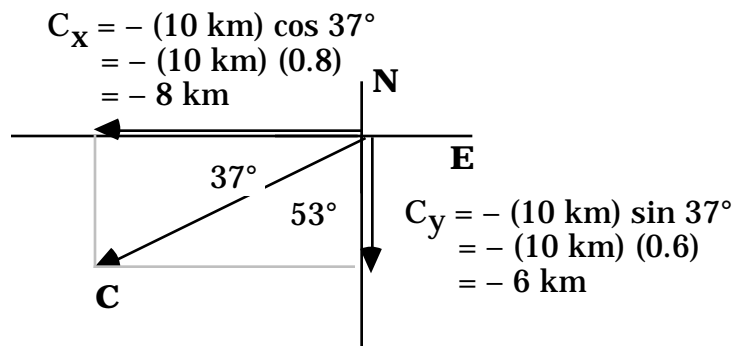
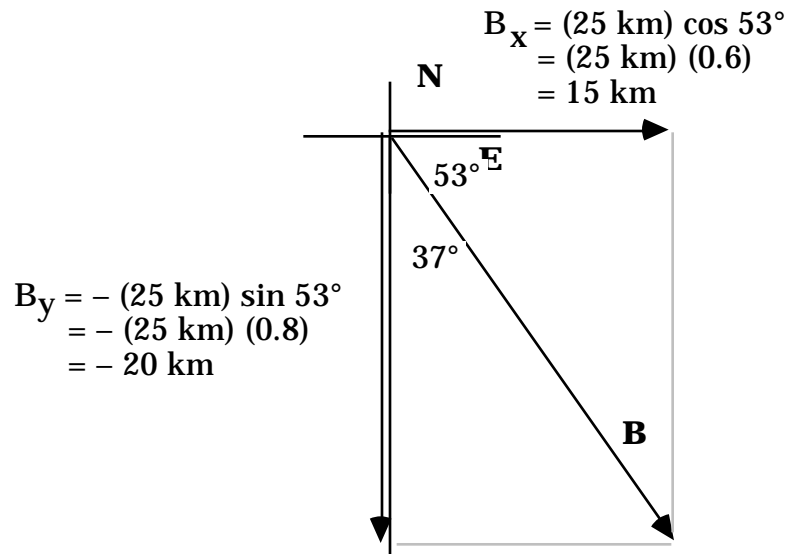
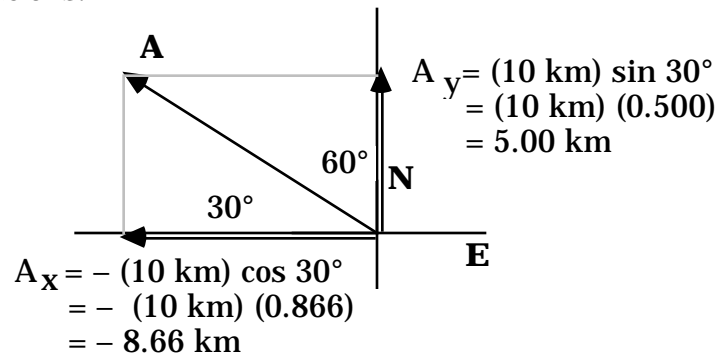
All four of the resultants  $\mathbf{R}$  are identical. The order in which we add vectors does not make any difference.

**3.15** **A** = 10 km, 60° W of N

**B** = 25 km, 37° E of S

**C** = 10 km, 53° W of S

As always, sketches are important. Draw these vectors first; only then should you begin your calculations.



**3.23** Now we want to **add** the components that we calculated in Problem 3.15.

$$A_x = -8.66 \text{ km}$$

$$A_y = 5.0 \text{ km}$$

$$B_x = 15 \text{ km}$$

$$B_y = -20 \text{ km}$$

$$C_x = -8 \text{ km}$$

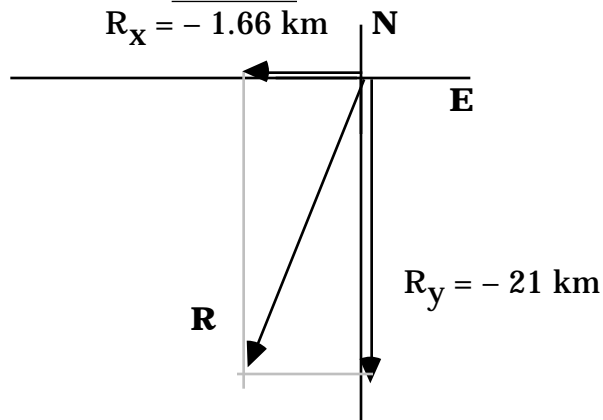
$$C_y = -6 \text{ km}$$

$$R_x = -1.66 \text{ km}$$

$$R_y = -21 \text{ km}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-1.66 \text{ km})^2 + (21 \text{ km})^2} = 21.1 \text{ km}$$

Now, what is the direction of the resultant?

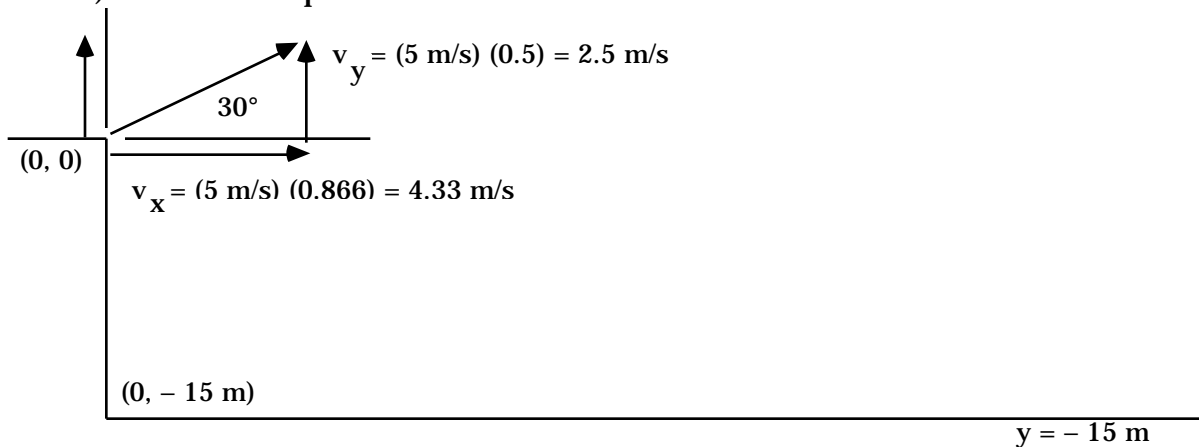


The angle can be found from  $\tan^{-1} \frac{\text{opp}}{\text{adj}} = \tan^{-1} \frac{1.66 \text{ km}}{21 \text{ km}} = 0.079; \quad = 4.5^\circ$ .

That is,  $\mathbf{R} = 21.1 \text{ km}, 4.5^\circ \text{ West of South}$ .

**3.35** A stone is thrown at an angle of  $30^\circ$  above the horizontal from the top of a cliff 15 meters above a wide river with an initial velocity of  $v_i = 5 \text{ m/s}$ .

- What are the horizontal and vertical components of its initial velocity?
- How long does it take to hit the water below?
- How far from the side of the cliff does it hit the water?
- What is its velocity when it hits the water?
- What is its speed when it hits the water?



**a)** What are the horizontal and vertical components of its initial velocity?

$$v_{x,i} = v_i \cos 30^\circ = (5 \text{ m/s}) (0.866) = 4.33 \text{ m/s}$$

$$v_{y,i} = v_i \sin 30^\circ = (5 \text{ m/s}) (0.500) = 2.50 \text{ m/s}$$

**b)** How **long** does it take to hit the water below?

At what time does the  $y$ -coordinate have a value of  $y = -15 \text{ m}$ ?

$$y = y_i + v_{y,i} t + (1/2) a_y t^2$$

$$-15 \text{ m} = 0 + (2.5 \text{ m/s}) t + (1/2) (-9.8 \text{ m/s}^2) t^2$$

$$-15 = 0 + 2.5 t - 4.9 t^2$$

$$4.9 t^2 - 2.5 t - 15 = 0$$

For  $a x^2 + b x + c = 0$ ,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Thus, with  $a = 4.9$ ,  $b = -2.5$ , and  $c = -15$ , we have

$$t = \frac{-(-2.5) \pm \sqrt{(-2.5)^2 - 4(4.9)(-15)}}{2(4.9)}$$

$$t = \frac{2.5 \pm \sqrt{6.25 + 294}}{9.8} = \frac{2.5 \pm \sqrt{300}}{9.8} = \frac{2.5 \pm 17.3}{9.8}$$

$$t = 2.02 \text{ s} \quad (t = -1.51 \text{ is the other, non-physical solution})$$

**c)** How far from the side of the cliff does it hit the water?

All this time (of 2.02 s) the stone has been traveling horizontally at a constant horizontal velocity of  $v_s = v_{x,i} = 4 \text{ m/s}$ . Therefore,

$$x = v_x t$$

$$x = (4 \text{ m/s}) (2.02 \text{ s})$$

$$\mathbf{x} = 8.08 \text{ m}$$

d) What is its velocity when it hits the water?

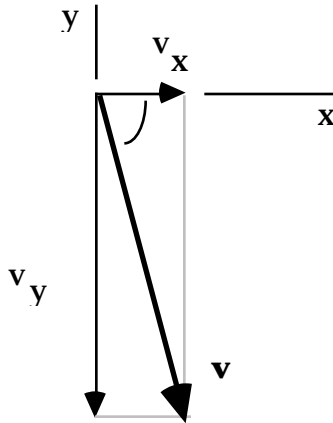
We already know the x-component of its velocity,  $\mathbf{v}_x = \mathbf{v}_{x,i} = 4.33 \text{ m/s}$ .

Its y-component of velocity is given by

$$v_y = v_{y,i} + a_y t$$

$$v_y = (2.5 \text{ m/s}) + (-9.8 \text{ m/s}^2) (2.02 \text{ s})$$

$$\mathbf{v}_y = -17.3 \text{ m/s}$$



The direction is given by

$$\begin{aligned} \tan \theta &= \frac{\text{opp}}{\text{adj}} = 17.3/4.33 = 4.0 \\ &= 76^\circ \end{aligned}$$

That is, we can say the velocity is

$$\mathbf{v} = 17.8 \text{ m/s at } 76^\circ \text{ below the horizontal.}$$

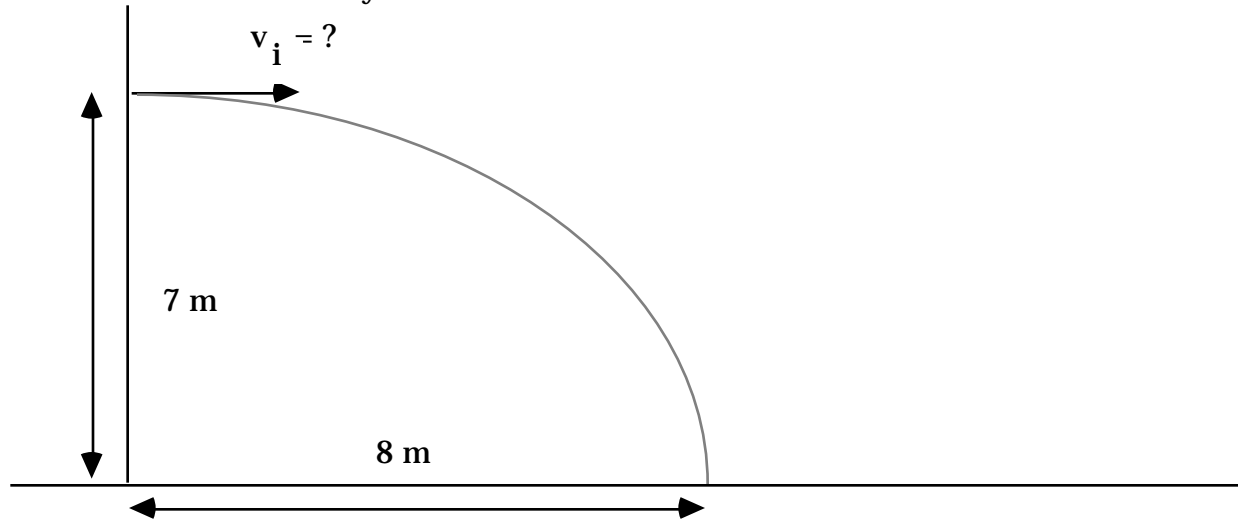
(where the magnitude 17.8 m/s comes from the next part of the question).

e) What is its speed when it hits the water?

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(4.33 \text{ m/s})^2 + (-17.3 \text{ m/s})^2} = \sqrt{318} \text{ m/s}$$

$$\mathbf{v} = 17.8 \text{ m/s}$$

**3.41** A book is thrown horizontally out a residence hall window 7.00 m above the grass outside the residence hall. The book lands on the grass 8.00 m from the base of the hall. Find the velocity with which the book was thrown.



Since the book initially moves horizontally, we immediately know  $v_{y,i} = 0$  and the vertical motion is simply free fall from rest. The time it takes the book to "fall" the 7.00 m to the ground can be found from

$$y = y_i + v_{y,i} t + \frac{1}{2} a_y t^2$$

We may as well take the ground to be  $y = 0$  and that makes the initial vertical position of the book  $y_i = +7.00$  m.

$$0 = 7.00 \text{ m} + 0 + \frac{1}{2} (-9.8 \text{ m/s}^2) t^2$$

$$4.9 t^2 = 7 \text{ s}^2$$

$$t^2 = 1.43 \text{ s}^2$$

$$t = 1.2 \text{ s}$$

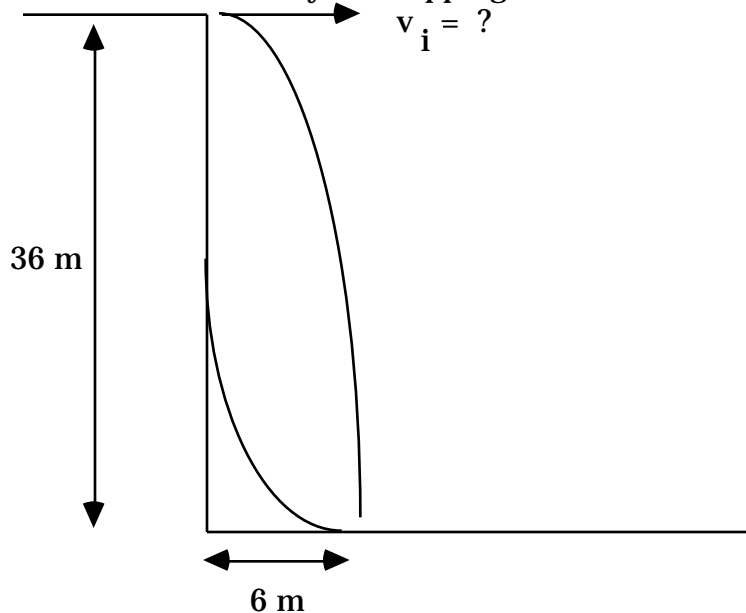
In that time of 1.2 s, the book travels horizontally a distance of 8 m so its horizontal velocity must be

$$v_x = \frac{8 \text{ m}}{1.2 \text{ s}} = 6.67 \text{ m/s}$$

The horizontal component of the velocity remains constant so

$$v_{x,i} = v_x = 6.67 \text{ m/s}$$

**3.43** Cliff divers at Acapulco jump into the sea from a cliff 36 meters high. At the level of the sea, there is an outcropping of rock that sticks out a horizontal distance of 6 meters. With what minimum horizontal velocity must the cliff divers leave the top of the cliff if they are to miss this rocky outcropping?



Assume the divers have an initial velocity that is horizontal—this means the initial vertical component of velocity is zero,  $v_{y,i} = 0$ . Then the time for the dive is given by

$$y = y_i + v_{y,i}t + \frac{1}{2} a_y t^2$$

$$0 = 36 \text{ m} + 0 + \frac{1}{2} (-9.8 \text{ m/s}^2) t^2$$

where we have taken the coordinate system to be such that  $y = 0$  at the sea and  $y_i = 36 \text{ m}$  at the cliff where the divers begin. Now we can solve for the time,  
 $(4.9 \text{ m/s}^2) t^2 = 36 \text{ m}$

$$t^2 = 7.35 \text{ s}^2$$

$$t = 2.7 \text{ s}$$

During that time of 2.7 s that the diver is in the air, he must move horizontally a distance  $x = 6 \text{ m}$  to avoid the rocks below. Since his horizontal velocity remains constant, that means

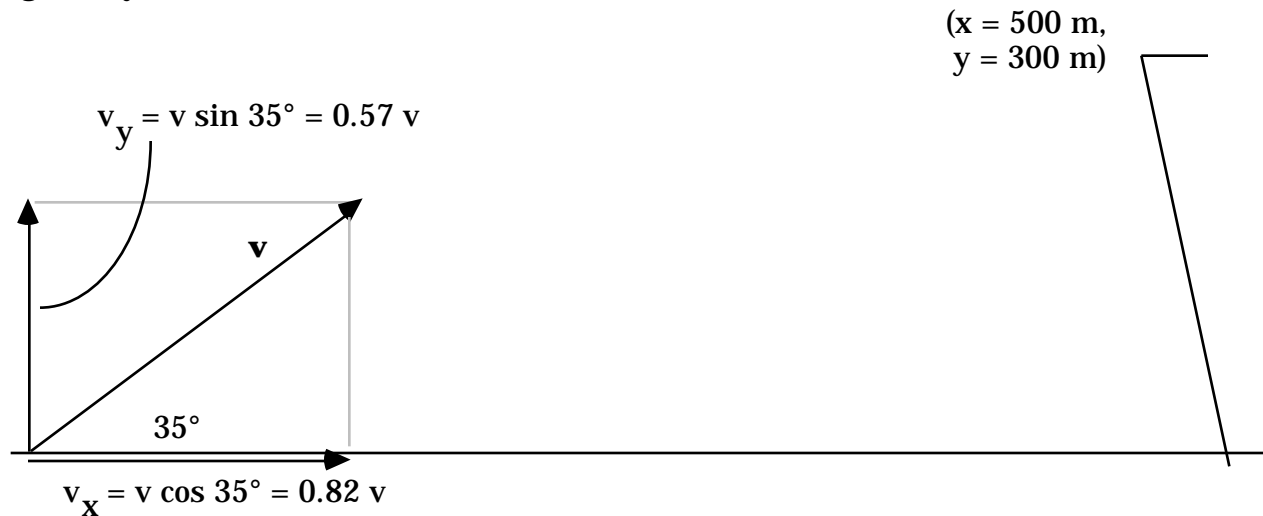
$$v_{x,i} = v_x = \frac{6 \text{ m}}{2.7 \text{ s}} = 2.2 \text{ m/s}$$

$$\mathbf{v_{x,i} = 2.2 \text{ m/s}}$$





**3.54** A mountain climber is stranded on a ledge 300 m above the ground. Rescuers on the ground want to shoot a projectile to him with a rope attached to it. If the projectile is directed upward at an initial angle of  $35^\circ$  and fired at the ledge from a horizontal distance of 500 m, determine the initial speed the projectile must have in order to land on the ledge. (Simply redrawing the diagram so the motion is from left to right may make it a little easier).



How long does it take the projectile to reach the cliff—that is, to have the horizontal position of the cliff?

$$x = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$x = 0 + (0.82 v) t + 0 = 500 \text{ m}$$

$$t = \frac{500 \text{ m}}{0.82 v}$$

In that amount of time, the vertical displacement of the projectile must equal the vertical displacement of the ledge, 300 m,

$$y = y_i + v_{y,i} t + \frac{1}{2} a_y t^2$$

$$300 \text{ m} = 0 + (0.57 v) \left( \frac{500 \text{ m}}{0.82 v} \right) + \frac{1}{2} (-9.8 \text{ m/s}^2) \left( \frac{500 \text{ m}}{0.82 v} \right)^2$$

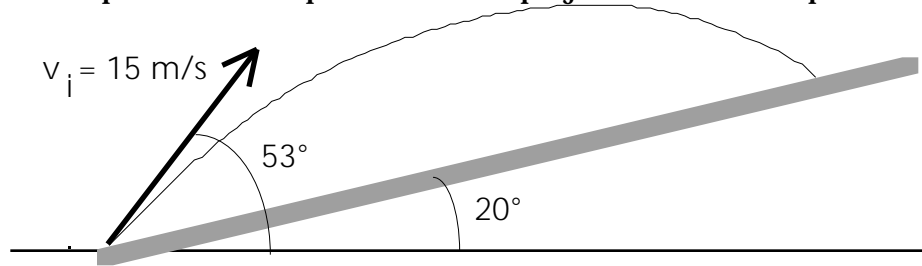
$$300 \text{ m} = 0 + 348 \text{ m} - 1,822,000 \frac{\text{m}^3}{\text{s}^2} \frac{1}{v^2}$$

$$-48 \text{ m} = -1,822,000 \frac{\text{m}^3}{\text{s}^2} \frac{1}{v^2}$$

$$v^2 = \frac{1,822,000 \text{ m}^2}{48 \text{ s}^2} = 37,950 \text{ s}^2$$

$$v = 195 \text{ m/s}$$

**3.67** A projectile is fired with initial velocity of 15 m/s at  $53^\circ$  above the horizontal from the foot of a plane inclined  $20^\circ$  above the horizontal as sketched in the figure below. How far up the inclined plane does the projectile strike the plane?



We need to find equations for the projectile's path and for the inclined plane and see where those two intersect (that is, where they have the same value).

The projectile moves along a path given by

$$x = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

and

$$y = y_i + v_{y,i} t + \frac{1}{2} a_y t^2$$

where  $v_{x,i} = (15 \text{ m/s}) \cos 53^\circ = (15 \text{ m/s}) (0.6) = 9 \text{ m/s}$

and  $v_{y,i} = (15 \text{ m/s}) \sin 53^\circ = (15 \text{ m/s}) (0.8) = 12 \text{ m/s}$

and  $a_x = 0$

and  $a_y = -9.8 \text{ m/s}^2$

Of course,  $x_i = 0$  and  $y_i = 0$ .

That is,

$$x = 0 + (9 \text{ m/s}) t + 0$$

and

$$y = 0 + (12 \text{ m/s}) t + \frac{1}{2} (-9.8 \text{ m/s}^2) t^2$$

From the x-equation, we may solve for t,

$$t = \frac{x}{9 \text{ m/s}}$$

and then substitute that into the y-equation,

$$y = 0 + (12 \text{ m/s}) \left( \frac{x}{9 \text{ m/s}} \right) + \frac{1}{2} (-9.8 \text{ m/s}^2) \left( \frac{x}{9 \text{ m/s}} \right)^2$$

$$y = 1.33 x - (0.605 \frac{1}{m}) x^2$$

If we are careful to measure both  $x$  and  $y$  only in meters, then we may drop the units and it may be easier to solve this problem if this equation is written as

$$y = 1.33 x - 0.605 x^2$$

Notice that this is a quadratic equation—the equation of a parabola curved downward. That is what we would expect.

Now, what is the equation of the inclined plane?

$$\tan 20^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

$$y = x \tan 20^\circ = 0.36 x$$

$$y = 0.36 x$$

Now we have two equations for  $y$ ; set those equal to each other,

$$y = 0.36 x = 1.33 x - 0.605 x^2$$

$$0.605 x^2 = 0.97 x$$

$$0.605 x = 0.97$$

$$x = \frac{0.97}{0.605} \text{ m} = 1.60 \text{ m}$$

$$y = 0.36 x = (0.36) (1.60 \text{ m}) = 0.58 \text{ m}$$

$$s = \sqrt{x^2 + y^2} = \sqrt{(1.6 \text{ m})^2 + (0.58 \text{ m})^2} = 1.70 \text{ m}$$

That is, the projectile strikes the inclined plane at **( $x = 1.6 \text{ m}$ ,  $y = 0.58 \text{ m}$ )** or at a distance of **1.70 m** along the plane.