Quiz 3

1) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < L$, subject to $u(0,t) = u(L,t) = 0$ for the following initial conditions:

a) $u(x,0) = \sin \frac{3\pi x}{L}$, $\frac{\partial u}{\partial t}(x,0) = \sin \frac{7\pi x}{L}$.

We use the formulas (4.4.12) on page 144 to determine the coefficients A_n and B_n.

Observe that $A_3 = 1$ and all other A_n's are zero. Also, $B_7 = \frac{L}{7\pi c}$ and all other B_n's are zero. So the solution is

$$u(x,t) = \sin \frac{3\pi x}{L} \cos \frac{3\pi ct}{L} + \frac{L}{7\pi c} \sin \frac{7\pi x}{L} \sin \frac{7\pi ct}{L}.$$

b) $u(x,0) = 0$, $\frac{\partial u}{\partial t}(x,0) = 1$.

First notice that all A_n's are zero. Then calulate the B_n's using $B_n \frac{n\pi c}{L} = \frac{2}{L} \int_0^L \sin \frac{n\pi x}{L} dx$.

$$B_n = \frac{L}{n\pi c} \begin{cases} 0, & \text{if } n \text{ is even} \\ \frac{4}{n\pi}, & \text{if } n \text{ is odd} \end{cases}$$

It now follows that

$$u(x,t) = \sum_{n=1}^{\infty} \frac{4L}{(2n-1)^2 \pi^2 c} \sin \frac{(2n-1)\pi x}{L} \sin \frac{(2n-1)\pi ct}{L}.$$