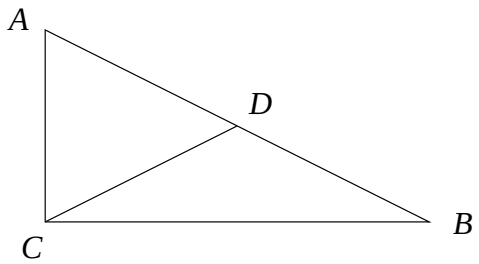
Challenges of the Week
Spring Semester 1996-1997

Challenge of the Week # 1 - January 24 to January 31:

1. *Is it possible to partition an arbitrary right triangle into isosceles triangles? (The triangles do not have to be congruent and there is no limit on the number of triangles you can use.) Justify your answer.*
2. *Can you partition an arbitrary triangle into isosceles triangle? Again justify your answer.*

From the paper of Brent Baker. It is possible to partition a right triangle into two isosceles triangles. Suppose $\triangle ABC$ is a right triangle with right angle at C. Since $\angle CAD$ is less than $\angle ACB$, it is possible to construct D such that $\angle ACD = \angle CAD$, as shown.



Since $\angle ACD + \angle DCB = 90^\circ = \angle CAD + \angle DBC$ and $\angle CAD = \angle ACD$, it follows that $\angle DCB = \angle DBC$. Hence $\triangle DBC$ is an isosceles triangle.

An arbitrary triangle can be partitioned into four isosceles triangles are follows. It is easy to see that if the altitude from a vertex of a triangle to the opposite side falls outside the triangle, then one of the two base angles is obtuse. Since a triangle has at most one obtuse angle, it is always possible to choose a vertex so that the altitude to the opposite side partitiions the triangles into two right triangles. By using the procedure outlined above, it is possible to partition each of these right triangles into two isosceles triangles.

Challenge of the Week # 2 - January 31 to February 7: A **disk** *of radius r is a circle of radius r together with its interior.*

1. *Show how to cover a disk of radius 2 with seven disks of radius 1.*
2. *What is the radius of the largest disk which can be covered with seven disks of radius 1? Prove your answer.*

Challenge of the Week # 3 - February 12 to February 21: Find the numerical value of FISH and SHOAL in the following "biological" problem. Different letters stand for different numbers in the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

$$FISH + FISH + \cdots + FISH = SHOAL.$$

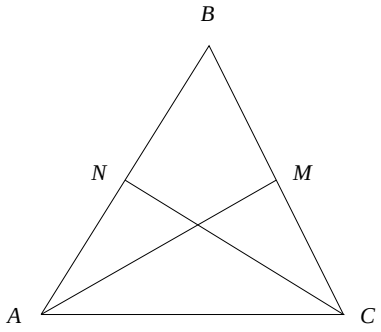
There are 73 FISH in this SHOAL

Challenge of the Week # 4 - February 21 to February 28: How many integers in the sequence

$$1,\ 11,\ 111,\ 1111,\ 11111,\ \ldots$$

are squares of integers? Justify your answer.

Challenge of the Week # 5 - February 28 to March 7: Suppose A, B, C are the vertices of a triangle, N is the midpoint of AB, M is the midpoint of BC, and $\angle BAM = \angle BCN = 30^\circ$, as indicated in the following diagram (which is not drawn to scale). Find the measure of $\angle ABC$. Prove your answer.



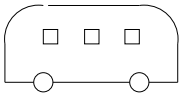
Challenge of the Week # 6 - March 7 to March 14: Find all ordered pairs (x, y) which satisfy the following system of simultaneous equations.

$$|x-3| = 5-y \quad (1)$$
$$x+y = |x-3|+|y-5| \quad (2)$$

Prove your answer.

Challenge of the Week # 7 - March 28 to April 4: This week's problems are a bit different in honor of April Fool's Day. Be sure to justify your answer to each problem.

1. The following picture is symmetric and shows a bus going forwards on its way from Charleston to Chicago. Which direction is the bus moving: from left to right or from right to left?



2. Move one digit in the following expression so that the equation becomes valid.

$$102 + 1 = 101$$

3. Two children, Dick and Jane, want to buy a book. However, Jane needs 99 more cents to buy the book, while Dick needs only one more cent. They decide to buy one copy of the book together, but discover they do not have enough money. What is the price of the book?

4. You have a pitcher of pure milk and a cup of pure coffee. You take a teaspoon of milk from the pitcher and put it into the cup. Without stirring too much, you take a teaspoon out of the non-uniform mixture in the cup and put it into the pitcher. Which is larger: the amount of milk in the cup or the amount of coffee in the pitcher?

Challenge of the Week # 8 - April 4 to April 11: Four hockey teams – A, B, C, and D – are to play each other once. After some of the matches have been played, a table giving some of the details of matches played, won, lost, etc., was as follows:

	Games Played	*Won*	*Lost*	*Drawn*	*Goals For*	*Goals Against*
A		*0*		*0*	*2*	*3*
B				*0*		*1*
C	*2*				*4*	
D		*0*		*1*		*5*

Find the score of each match. Justify your answer.

Challenge of the Week # 9 - April 11 to April 18: Recently an ancient tablet was discovered which shows that soccer was played thousands of years ago. The tablet shows the results of a partially completed 5 team tournament in which each team has to play each of the others exactly once. (The missing figures were apparently obliterated.)

Team	*Games Played*	*Won*	*Lost*	*Drawn*	*Goals For*	*Goals Against*	*Points*
A	*3*				*0*	*1*	*1*
B	*3*				*3*	*1*	*4*
C	*3*				*1*		*3*
D	*3*			*2*	*1*		*3*
E	*3*			*0*			*2*

(Two points are awarded for a win, one point for a drawn (to each team), and zero points for a loss.) Historical research has shown that: D had already played B, A had played E, B did not have two draws, that the highest scoring match was the one between B and E, and that the above table contains **exactly two mistakes.**

Determine the score of every match played thus far. Justify your answer.

Challenge of the Week # 10 - April 18 to April 25: Suppose ABCD is a quadrilateral each of whose sides is less than one. Show that the area of ABCD is less than one.