# Challenges of the Week
## Spring Semester 1995-96

*Challenge of the Week # 1 - January 19 to January 26: Initially there is one empty box. Into that box, $k$ (smaller) empty boxes are inserted. The initial box is no longer empty. Next, insert $k$ empty boxes into any empty box. The process (inserting $k$ empty boxes into any box that is empty) is repeated until there are exactly $m$ non-empty boxes. At this time, how many empty boxes are there, in terms of $k$ and $m$?*

*Challenge of the Week # 2 - 2 to Friday: February 2*

*Challenge of the Week # 3 - February 2 to February 9:*

a) *Change some of the $\pm$ signs to $+$ and the rest to $-$ in the expression*

$$\pm 1 \pm 2 \pm 3 \pm 4 \pm \cdots \pm 95 \pm 96$$

*to obtain an expression involving 96 integers and 96 signs which equals 1996.*

b) *Determine the maximum number of $+$ signs that can be used in such an expression which equals 1996. Justify your answer.*

*Challenge of the Week # 4 - February 16 to February 23: Three joggers, Ann, Betty, and Cynthia, jogged along a circular path. Each jogger ran with a constant speed but Betty ran faster than Cynthia and slower than Ann. They started at the same moment at the same point and finished as soon as all three were again at the same point (which was not necessarily the point where they started.) It turned out that Ann passed Cynthia 14 times. Find the total number of passings during the run. Justify your answer.*

*Challenge of the Week # 5 - February 23 to March 1: The functions $f$ and $g$ are defined by $f(x) = \sqrt{x^3+1}$ and $g(x) = \sqrt[3]{x^2-1}$, respectively.*

1. *Verify that $g(f(x)) = x$ for any $x \geq 0$ and $f(g(x)) = x$ for $x \geq 1$.*
2. *Show that*
$$\int_0^2 f(x)\ dx + \int_1^3 g(x)\ dx = 6.$$

*Challenge of the Week # 6 - March 1 to March 8: In a room there are 100 people whose ages are*

$$a_1, a_2, a_3, \ldots, a_{100},$$

*where each $a_i$ is a positive integer not more than 99.*

1. *Show there are at least two people who have the same age.*
2. *Show it is possible to find $m$ people, for some $m \leq 100$, so that the sum of the ages of these $m$ people is divisible by 100.*

*Challenge of the Week # 7 - March 8 to March 15:*

1. *Is it possible to find natural numbers $a, b, c$ and $d$ so that the sums $a+b$, $a+c$, $a+d$, $b+c$, $b+d$, and $c+d$ represent (not necessarily in that order) six consecutive integers?*
2. *Is it possible to find natural numbers $a, b, c, d$ and $e$ so that the sums $a+b$, $a+c$, $a+d$, $a+e$, $b+c$, $b+d$, $b+e$, $c+d$, $c+e$, and $d+e$ represent (not necessarily in that order) ten consecutive integers?*

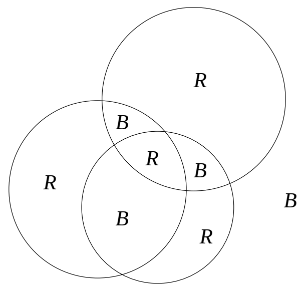
*Justify your answers.*

*Challenge of the Week # 8 - March 29 to April 5: Given four points $A$, $B$, $C$, and $D$ in the plane, one can measure six distances between them: $AB$, $AC$, $AD$, $BC$, $BD$, $CD$.*

1. *Is it possible that these six distances (not necessarily in the same order as above) are 1 cm, 2 cm, 3 cm, 4 cm, 5 cm, and 6 cm?*
2. *Is it possible that five of these distances are equal to 10 cm and the sixth distance is 17 cm?*

*Justify your answers.*

*Challenge of the Week # 9 - April 5 to April 12: Some circles are drawn on the plane. Some intersect but none are tangent and it is never the case that three or more circles meet at the same point. These circles divide the plane up into a 'map'. Show that you can color the 'countries' of this map in two colors so that no two 'countries' which share a common edge are colored with the same color. An example with 3 circles is given below. A coloring by two colors, R and B, is also indicated.*




*Challenge of the Week # 10 - April 12 to April 19: The three sides of a right triangle have lengths a, b, and c, with $a^2 + b^2 = c^2$. Further, a, b, and c are integers.*

1. *Show that ab is even.*
2. *Show that the area of the triangle is divisible by 6.*

*Challenge of the Week # 11 - April 19 to April 26: The following set of bicycle tracks were recently noticed in a muddy alleyway in Charleston. Which way was bicycle going and which track is the track of the front wheel? Justify your answers.*