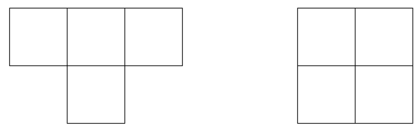
Challenges of the Week
Spring Semester 1993-1994

Challenge of the Week # 1 - January 21 to January 28: *Can an ordinary 8×8 checkerboard be covered (without overlapping) by 16 tiles of shape A (shown below)?*
Can the checkerboard be covered by 15 tiles of shape A and 1 tile of shape B? Justify your answers.



A B

Challenge of the Week # 2 - January 31 to February 4: *You are given three containers – one containing 8 liters, one containing 5 liters and one containing 3 liters. The 8 liter container is filled with water. Using only these three containers show how to divide the water into two 4 liter portions.*

Challenge of the Week # 3 - February 4 to February 14: *On each of three cards is written a whole number from 1 to 10. These cards are shuffled and dealt to three people who record the numbers on the card they are dealt. The cards are collected and the shuffling-dealing-recording process is repeated several times. The three people then find the sums of the numbers on the cards they received and the sums are found to be 13, 15, and 22. What are the numbers on the cards? Justify your answer.*

Challenge of the Week # 4 - February 18 to February 25: *In the multiplication below, each letter represents a different digit*

$$\begin{array}{cccc} & & & A \\ \times & & C & B \\ \hline & & E & D \\ & G & F & \\ \hline & D & D & D \end{array}$$

Which of the ten digits does D represent? Justify your answer.

Challenge of the Week # 5 - February 25 to March 4: *Find the smallest positive integer n such that every integer greater than or equal to n can be written as a sum of fives and eights. Justify your answer.*

Challenge of the Week # 6 - March 4 to March 11: *Show that given five integers, not necessarily distinct, it is always possible to find three of these integers whose sum is divisible by three.*

Challenge of the Week # 7 - March 11 to March 18: *Given any five points in a unit square, prove there are at least two which are within $\sqrt{2}/2$ of each other.*

Challenge of the Week # 8 - April 1 to April 8: *What is the largest number of pieces which can be placed on an 8×8 checkerboard so that no more than 2 pieces are in a straight line in* any *direction? All pieces are assumed to be placed at the center of a playing square. Justify your answer.*

Challenge of the Week # 9 - April 8 to April 15: *A penny is placed in the middle of each square of a 5×5 "checkerboard". Initially there are more tails than heads. A move consists of turning over each coin in a single row or a single column. Show it is possible to make a sequence of moves so that every row and column contains more heads than tails.*

Challenge of the Week # 10 - April 15 to April 22: *Let a, b, and c be the lengths of the sides of a triangle. Show that if $a^2 + b^2 + c^2 = ab + ac + bc$, then the triangle is equilateral.*

Challenge of the Week # 11 - April 22 to April 29: *In the number 234,765, the two "halves" 234 and 765 add up to 999. Also, 234,765 is evenly divisible by 999. Is this just a coincidence? Justify your answer.*