

Solution - Challenge of the Week

Challenge of the Week # 6 - February 22 to February 29, 2007

A person is in the middle of round pool which is 200 feet in diameter. A very vicious dog is running around the outside of the pool, but will not enter the pool because it is afraid of the water. The dog can run around half of the circumference of the pool in the same time that it takes for the person to swim half of the diameter of the pool. In order to escape, the person must reach the edge of the pool before the dog gets there. Can the person escape, or not? Justify your answer.

There were no correct solutions given to this week's challenge. However, Julee Simpson and Peter Provin submitted attempts.

The person swims out one-quarter of the radius of the pool. The person then swims around the circle, centered at the center of the pool, until he/she is directly opposite the dog, at which time the person swims to the closest point on the edge of the pool.

To see that this method will work, let a be the speed of the person, b be the speed of the dog, and r the radius of the pool. The time required for the person to swim the radius of the pool is r/a . The time required for the dog to run around half of the circumference of the pool is $r\pi/b$. Since these are equal $a = b/\pi$.

The time required for the person to swim around a circle of radius $r/4$ is

$$\frac{2\pi(\frac{r}{4})}{a} = \frac{r\pi}{2a} = \frac{r\pi^2}{2b}.$$

The time required for the dog to run around the pool is $\frac{2r\pi}{b}$. Since $\pi < 4$,

$$\frac{r\pi^2}{2b} < \frac{4r\pi}{2b} = \frac{2r\pi}{b}.$$

Therefore, the person can swim around the small circle faster than the dog can run around the pool. This means that the person can always swim around the circle to obtain the situation where he/she, the center of the pool and the dog are in a straight line with the center of the circle in the middle. At this time, the distance from the person to the closest point on the radius of the pool is less than the radius of the pool while the distance that the dog must run to get to that point is one-half of the circumference of the pool. Therefore, the person can escape.