Solution - Challenge of the Week

Challenge of the Week # 2 - January 18 to January 25, 2007

The two people from last week's challenge are still playing games.

(a) One coin is placed in the middle of a large rectangular table. Now the two people take turns placing coins on the table. The coins do not overlap and each lies entirely on the table. The player who places the last coin on the table wins. Can one of the players always win? If so, what is her/his winning strategy.

(b) They change the rules of the game again. This game is the same as the last game, except that, before play begins, a coin is placed on the table, but **not** in the middle of the table. Who wins this game? If one player can always win, what is the winning strategy?

There was a submission from Lindsay Harper, but no correct solution were received for these challenges.

(a) The second player can always win this game. Let a coordinate system be introduced so that the origin is at the center and the axes run parallel to the edges of the table. Because of the coin in the center of the table, every possible position at which a coin can be placed is centrally symmetric to another point at which a coin can be placed. If the first player places a coin at, say (x, y), the second player can place a coin at $(-x, -y)$ and leave the coins on the table in a symmetrical position. Thus, every move by the first player destroys the symmetry, but the second player can always restore the symmetry on his/her next move. Thus the second player will win, since she/he will always be able to move.

(b) There probably isn't a strategy for this game since the moves depend so much upon the size of the table and the size of the coin. Because the initial position is not symmetric, the winner depends upon how many coins will fit on the table. If the coins on the table are centrally symmetric, with the origin occupied, and if there is a position on the table for one coin, then there is a centrally symmetric position at which a second coin can be placed. Further, after the placement of this coin, the coins on the table are again centrally symmetric. To see the difficulty involved if there is no central symmetry, take a square whose edges are slightly more than $1\frac{3}{4}$". Three quarters will fit in the square, but it also possible to place two quarters in the square so that a third quarter cannot be placed in the square.