Challenges of the Week
Spring Semester 2007-2008

Challenge of the Week # 1 - January 11 to January 18:

(a) *Two people are playing a game. The players take turns placing coins on a large rectangular table. The coins may not overlap and they must lie entirely on the table. The player who places the last coin on the table wins. Who wins? And, if one of the players can always win with the appropriate strategy, what is the winning strategy?*

(b) *After this game, the two people go to a different game. They take turns placing a circular magnets on a very large metal, spherical ball. With the same rules for winning as in the previous game, who wins? What is the winning strategy, if there is one?*

Challenge of the Week # 2 - January 18 to January 25: The two people from last week's challenge are still playing games.

(a) *One coin is placed in the middle of a large rectangular table. Now the two people take turns placing coins on the table. The coins do not overlap and each lies entirely on the table. The player who places the last coin on the table wins. Can one of the players always win? If so, what is her/his winning strategy.*

(b) *They change the rules of the game again. This game is the same as the last game, except that, before play begins, a coin is placed on the table, but* ***not*** *in the middle of the table. Who wins this game? If one player can always win, what is the winning strategy?*

Challenge of the Week # 3 - January 25 to February 1: Find all three digit numbers, n, which have the following property: The three rightmost digits of its square, n^2, form the same integer n. Justify your answer.

Challenge of the Week # 4 - February 1 to February 8: Let a, b, and c be three unknown real numbers which are not integers. Let

$$n = a - b + 2008, m = b - c + 2008, k = c - a + 2008.$$

Suppose that m, n and k are three consecutive integers. Find the three integers. Justify your answer.

Challenge of the Week # 5 - February 8 to February 22: Since there is no class on February 15, this will be a "Challenge of the Two Weeks".
The sum of two positive integers is 2008. Suppose that if you divide the larger integer by the smaller one, you get a positive remainder r. What is the largest possible value of r? Justify your answer.

Challenge of the Week # 6 - February 22 to February 29: A person is in the middle of round pool which is 200 feet in diameter. A very vicious dog is running around the outside of the pool, but will not enter the pool because it is afraid of the water. The dog can run around half of the circumference of the pool in the same time that it takes for the person to swim half of the diameter of the pool. In order to escape, the person must reach the edge of the pool before the dog gets there. Can the person escape, or not? Justify your answer.

Challenge of the Week # 7 - February 29 to March 7: Does there exist a positive integer n, whose square equals

$$7^{198} + (2^{200} \cdot 7^{99}) + 2^{398}?$$

If yes, is n a prime number of a composite number? Justify your answer

Challenge of the Week # 8 - March 21 to March 28: A cubical box contains some marbles. Each of the six faces of the box is labelled with a different number chosen from 1, 2, 3, 4, 5, 6. Also, each face contains the following statement:

The number of marbles in this box is greater than $10 + 2k$,
where k is the label on this face.

If half of the statements are true and half are false, how many marbles are in the box? Justify your answer.

Challenge of the Week # 9 - March 28 to April 4:

This is the annual April Fool's edition of the Challenge of the Week.

1. *One person is standing on the balcony. On the ground immediately below, another person is standing. The two people shout "Hey" at exactly the same time. Who hears the other person's shout first, the person on the ground or the person on the balcony?*
2. *Find the two leftmost digits of the following number:*

$$1^1 + 2^2 + 3^3 + \cdots + 999^{999} + 1000^{1000}.$$

3. *Put one mathematical sign between the digits* 5 *and* 6 *to get a number between 5 and 6. (Note:* $5-6$, $5\sqrt{6}$, *and* $5/6$ *will not work.)*
4. *Are there two different sequences of 9 consecutive integers such that the product of the integers in one sequence equals the product of the integers in the other sequence? How many pairs of such sequences are there?*

In each case, justify your answer.

Challenge of the Week # 10 - April 4 to April 11: Two trains run on parallel tracks, one train is 1200 feet long and the other train is 1500 feet long. Each train runs at a constant speed. When they are going in the same direction, it takes 75 seconds for one train to completely overtake the other. When they are going in opposite directions, it is only 15 seconds between time that the heads of the two trains are even to when the ends of the trains are even. Determine the speed of the longer train. Note: There could be two answers!