

Challenges of the Week Spring Semester 2006-2007

Challenge of the Week # 1 - January 12 to January 19: Find all positive integers which, when written in base 10, have all of the following properties:

- (a) The digits of the integer, when added together, give a sum of 100,*
- (b) the squares of the digits of the integer, when added together, give a sum of 100, and*
- (c) no two consecutive digits of the integer are equal.*

Justify your answer.

Challenge of the Week # 2 - January 19 to January 26: There are two neighboring countries, Astoria and Burea. There are direct airline flights between some of the cities of Astoria and some of the cities of Burea. A city in either country is called a "big city" if it is connected by a direct flight to more than half of the cities in the other country. All other cities in the two countries are called "small cities".

Suppose in Astoria, there is a small city which is connected by direct flights to all of the small cities of Burea. Show that every big city in Astoria is connected by a direct flight to at least one big city in Burea.

Challenge of the Week # 3 - January 26 to February 2: Do there exist two triangles, T_1 and T_2 , such that

- (a) they are not congruent, but*
- (b) for every side of T_1 , there is a side of T_2 which has the same length and for every side of T_2 , there is a side of T_1 which has the same length?*

Justify your answer.

Challenge of the Week # 4 - February 2 to February 9: Do there exist two triangles, T_1 and T_2 , such that

- (a) they are not congruent, but*
- (b) for **every pair** of sides of T_1 , there is a pair of sides of T_2 such that the sum of lengths of the two sides from T_1 is the same as the sum of the lengths of the two sides from T_2 ?*

Justify your answer.

Challenge of the Week # 5 - February 14 to February 23: In a small forest there are exactly 300 trees. The trees in the forest are either maple trees, oak trees, or chestnut trees. It is the case that no matter how you select 201 different trees from the forest, at least one of 201 will be a maple tree, at least one will be an oak, and at least one will be a chestnut. Rodney, the Ranger, claims that there are exactly 100 oak, 100 maple, and 100 chestnut trees in the forest. Is he correct? Justify your answer.

Challenge of the Week # 6 - February 23 to March 2: A 19-digit positive integer, N , has the property that it is divisible by 11 and whenever two non-equal digits of N are interchanged, the resulting number is not divisible by 11. Find the smallest possible N with these properties. Justify your answer.

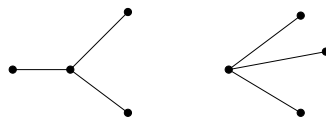
Challenge of the Week # 7 - March 23 to March 30: At a certain company, some of the people shake hands with others. No one shakes hands with a person more than once. And no one shakes hands with herself or himself. It is discovered that there is one person who made exactly one handshake, one who made exactly two handshakes, one who made exactly three hand shakes, and so on until one who made exactly 10 handshakes. Determine the smallest possible number of people at this company. Justify your answer.

Challenge of the Week # 8 - March 30 to April 6: There are fewer than 40 students in a certain grade school class. The average weight of all the students in the class is 71 pounds. The average weight of the boys is 84 pounds, while the average weight of the girls is only 54 pounds. How many boys and how many girls are in the class? Justify your answer

Challenge of the Week # 9 - April 6 to April 13: Ten quarters are placed into two flexible coin purses. The first purse contains A quarters and the second contains B quarters. Can it happen that the ratio A to B equals 10 to 1? Can it equal 5 to 1? Can it equal 2 to 1? Justify your answers.

Challenge of the Week # 10 - April 13 to April 20: In a city, there are six parks. There are streets, **which are segments of straight lines**, between some of the parks. The following conditions are satisfied:

- (a) Each park is connected by straight line streets with exactly three other parks.
- (b) None of the straight line streets intersect each other.
- (c) For every park, the three streets which meet there have the property that two of the streets form an angle, less than 180° , which contains the other street. Thus, the first arrangement of streets, shown below, does not occur. The second arrangement can occur.



Draw a possible plan for the six parks and their connecting streets.