Challenges of the Week
Spring Semester 2004-2005

Challenge of the Week # 1 - January 21 to January 28: Four persons, A, B, C, and D, are playing poker. At some time during the game; A has as many chips at B, C, and D, together; B has half as many chips as A, C, and D, together; C has 1/N the total number of chips of A, B, and D, together; and D has 1/M the total number of chips of A, B, and C, together; where M and N are different positive integers.

1. *How many poker chips do the four players have together? (There are many possible answers.)*
2. *What is the fewest number of chips that the four players together could have?*

Justify your answers.

Challenge of the Week # 2 - January 28 to February 4: John has 5 slices of pizza and Mary has 3 slices. Robert, who is very hungry, arrives and says that he will pay \$8, if they will divide the pizza equally among the three of them. They agree, so Robert divides the \$8 between John and Mary so they each receive the same amount of money per slice for the pizza they sold to him. How much money did Robert give to Mary? How much money did Robert give to John? Justify your solution.

Challenge of the Week # 3 - February 4 to February 11: An triangle isosceles is cut into two smaller isosceles triangles with a single straight cut. What are the possible values for the angles of the triangle? Justify your answer.

Challenge of the Week # 4 - February 11 to February 25: Let N be a positive integer. Consider all of the digits that occur in either the decimal expansion of N or the decimal expansion of $3N$ or in both expansions. Show that one of these digits is either 1, 2 or 9.

Challenge of the Week # 5 - February 25 to March 4: I recently got lost in a very large city. All of the streets in this city go either north-south or east-west. It seemed like I was walking forever. I started down one street and wandered around making left and right turns at random, never going down the same street twice and never crossing my path, until I eventually returned to the starting point, going in the same direction that I started out. I counted. I made a total of 100 left turns in my journey. How many right turns could I have made? Justify your answer.

Challenge of the Week # 6 - March 4 to March 11: The positive integer N can be doubled if the last digit in its decimal expansion is moved to the front of its decimal expansion. The last digit of N is 2. What is the smallest possible value of N. Justify your answer.

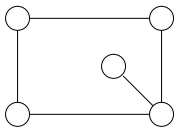
Challenge of the Week # 7 - March 25 to April 1: Does there exist a right triangle $\triangle ABC$ with $\angle B = 90^\circ$, base $AC = 10$, and area 30? Justify your answer.

Challenge of the Week # 8 - April 1 to April 8: This is the annual April Fool's Day edition of the Challenge of the Week. Justify each of your answers.

1. *Ten is to Three as Three is to Five as Five is to Four as Four is to Four as Thirteen is to ???*
2. *There are three volumes in a certain trilogy. Each volume has 400 pages. The volumes are arranged in order on a shelf. A worm has eaten from the first page of the first volume to the last page of the third volume. How many pages has the worm eaten through?*
3. *You have a pile of 100 bricks. You need to count out 70 bricks. The bricks are heavy so it takes 1 second to count each brick. Can you count out 70 bricks in less than 1 minute? Can you count out 70 bricks in less than 45 seconds?*
4. *Salvador Dali, the Spanish artist who lived from 1904 to 1989, wrote in his book "The Secret Life of Salvador Dali", "Before the contest I bet that I'll win the contest by drawing a picture and never touching a brush or a pencil to the canvas — and I will always receive first prize". What was Salvador Dali's secret method of drawing?*

Challenge of the Week # 9 - April 9 to April 15: At a certain very fancy store they sell expensive letters to be used in spelling out numbers. The letters are priced individually. Different letters of the alphabet may have different prices, but different copies of the same letter cost the same. It costs \$6 for the letters of "ONE", \$9 for the letters of "TWO", and \$16 for the letters of "ELEVEN". How much does it cost for the letters of "TWELVE"?

Challenge of the Week # 10 - April 15 to April 22: Five different circles are connected as shown:



1. *How many different color patterns can be obtained if three different colors must be used and no two adjacent circles may be of the same color? Justify your answer.*
2. *How many different color patterns can be obtained if four different colors must be used and no two adjacent circles may be of the same color? Justify your answer.*