Challenges of the Week
Spring Semester 2001-2002

Challenge of the Week # 1 - January 11 to January 18: Among grandfather's papers a bill was found

72 turkeys $ __67.9__

The first and last digits of the number that obviously represented the total price of these turkeys are replaced by blanks because they had faded and are now illegible.

What are the two faded digits and what was the price of one turkey? Justify your answer.

Challenge of the Week # 2 - January 18 to January 25: There are five people sitting in a circle - Andy, Betty, Cathy, Dougie, and Evie. Each is wearing a hat, which is either red or white. Each of person can see all hats except his or her own. The people make the following, truthful, statements:

Andy: I see exactly three red hats.

Betty: I see exactly three red hats.

Cathy: I also see exactly three red hats.

What color of hat is Andy wearing? Justify your answer.

Challenge of the Week # 3 - January 25 to February 1:

1. *Do there exist positive integers, x_1, x_2, $\ldots x_n$, such that both*

$$x_1 + x_2 + \cdots + x_n = 2002$$

and

$$x_1 \times x_2 \times x_2 \times \cdots \times x_n = 2002?$$

Either provide an example to show such integers exist or show that no such integers exist.

2. *Do there exist positive integers, x_1, x_2, $\ldots x_n$, such that both*

$$x_1 + x_2 + \cdots + x_n = 2003$$

and

$$x_1 \times x_2 \times x_2 \times \cdots \times x_n = 2003?$$

Either provide an example to show such integers exist or show that no such integers exist.

Challenge of the Week # 4 - February 4 to February 15: In the following problems, blanks do not count as either letters or spaces.

1. *If it is possible, replace the blank in the sentence below with a number, written out in words (like fifty five) so that the sentence is true. If it is impossible, explain why that is the case.*

This sentence contains ________________________ letters.

2. *If it is possible, replace the blank in the sentence below with a number, written out in words (like fifty five) so that the sentence is true. If it is impossible, explain why that that is the case.*

This sentence contains ________________________ characters.

Challenge of the Week # 5 - February 15 to February 22: An **infinite, geometric sequence** *is an infinite sequence where each term is a fixed multiple of the preceding term. For example,*

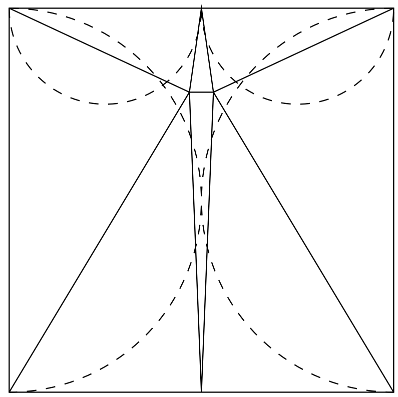
$$2, 4, 8, 16, 32, 64, 128, \ldots$$

is a geometric sequence as is

$$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$$

Does there exist a geometric sequence in which the first 10 terms are integers and the other terms are not integers..? Justify your answer.

Challenge of the Week # 6 - February 22 to March 1: Recall that a triangle is said to be **acute** *if all of its angles are less than* 90° *and is said to be* **obtuse** *if one of its angles is more than* 90°. *The following diagram shows how a square can be divided into triangles so that all of the triangles are acute. Note: The curved, dotted lines below are semi-circles. Using the result that in a triangle, say* $\triangle ABC$, *the angle* $\angle ACB$ *is acute if and only if* C *is outside the circle with diameter* AB, *we see that all triangles in the diagram are indeed acute.*



Show how to divide a square into triangles so that all of them are obtuse. The award will be given to the solution involving the fewest number of triangles.

Challenge of the Week # 7 - March 1 to March 8: An **infinite, geometric sequence** *is an infinite sequence where each term is a fixed multiple of the preceding term. For example,*

$$2, 4, 8, 16, 32, 64, 128, \ldots$$

is a geometric sequence as is

$$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$$

Challenge of the Week #5 was to find, if possible, an infinite, geometric sequence in which the first 10 terms are integers and the other terms are not integers. This week's challenge is to find an infinite, geometric sequence in which the first 20 terms are integers, the other terms are not integers, and **such that the sequence is increasing**. *Justify your answer.*

Challenge of the Week # 8 - March 22 to March 29: The sum of the digits of 3^{16} *is 27 and the sum of the digits of* 3^{17} *is also 27. Does there exists a positive integer,* n, *such that the sum of the digits of* 2^n *is the same as the sum of the digits of* 2^{n+1}? *Justify your answer.*

Challenge of the Week # 9 - March 29 to April 5: This is the annual April Fool's edition of the Challenge of the Week:

1. *A caterpillar is climbing up a 10 foot tall tree. Every day it climbs up 5 feet, but every night, when it sleeps, it slides down 4 feet. At this rate, how many days and nights will it take for the caterpillar to get to the top of the tree?*
2. *Suppose* P *is a point in space and* $\mathcal{C}$ *is a curve which starts at* P, *never intersects itself, and is such that every point on* $\mathcal{C}$ *is at most 1 foot from* P. *Is it possible that* $\mathcal{C}$ *can be 2002 feet long?*
3. *A person 6 feet tall walks around the earth at the equator (assume the person can walk on water). The top of the head of the person travels further than the soles of her feet on this trip. How much further?*
4. *A survey was conducted recently in which the number of hairs on the head of each citizen of New York City was counted. These numbers were all recorded in a huge spreadsheet. What is larger, the sum of these numbers or the product of these numbers?*

Justify your answer in each case. The award(s) will be given to the submission(s) with the most correct answers.

Challenge of the Week # 10 - April 5 to April 12: A company sells candy bars, each of which has a coupon enclosed. You can buy one of these candy bars for 99 cents. You can redeem nine coupons for another candy bar, which, of course, will have a coupon enclosed. What is the actual cost of each candy bar?