Challenges of the Week
Fall Semester 1996-1997

Challenge of the Week # 1 - August 28 to September 6: You are given four congruent right circular cones (about the size of ice cream cones). These are placed on a table. There are six distances between the pairs of vertices of cones. Is it possible to place the cones on the table so that these six distances are equal?

Challenge of the Week # 2 - September 6 to September 13: Let p_1 *and* p_2 *be consecutive odd primes (for example, 13 and 17 are consecutive odd primes). Show that the sum* $p_1 + p_2$ *can be factored as the product of three integers, all larger than 1. (Of course, solving this problem for a specific case is not sufficient.)*

Challenge of the Week # 3 - September 13 to September 20: Find two six digit numbers p_1 *and* p_2 *such that* $p_1 \neq p_2$ *and when* p_1 *and* p_2 *are placed next to each other the result is a 12 digit number which is divisible by* p_1p_2

Challenge of the Week # 4 - September 20 to September 27: You are given n *identical candles and a candelabra holding* n *candles. The candles are to be burned in the following way: on the first day one candle is burnt for one hour, on the second day two candles are burnt for one hour, ..., and on the* n*-th day all candles are burnt for one hour. Finally, the candles are burnt in such a way that all burn down at the same time. For which values of* n *is this possible? Justify your answer, explaining how it can be done when possible.*

Challenge of the Week # 5 - September 27 to October 4: Find all complex numbers a *and* b *such that*

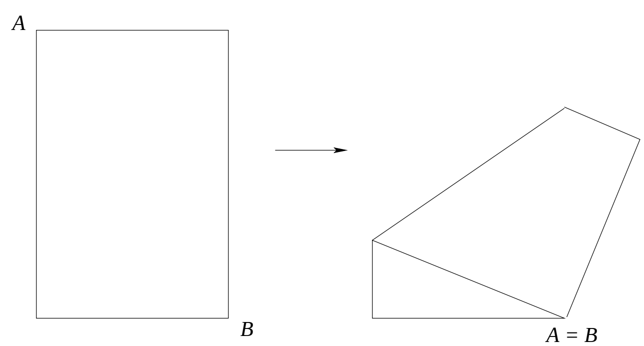
$$ab = a + b = \frac{a}{b}.$$

Justify your answer.

Challenge of the Week # 6 - October 4 to October 11: Find all 3-digit numbers m *which are equal to the arithmetic mean of the six numbers which can be obtained by rearranging the digits of* m *in all possible ways.*

Challenge of the Week # 7 - October 11 to October 18: Initially you have one liter of liquid in a green container and an empty, white, one liter container. You first pour one-half of the liquid in the green container into the white container, then you pour one-third of the liquid in the white container into the green container, then you pour one-fourth of the liquid in the green container into the white container, . . . , and finally pour 1/1997-th of the liquid in the white container into the green container. How much liquid is left in the green container? Justify your answer.

Challenge of the Week # 8 - October 18 to October 25: A rectangular piece of paper is folded so that two diagonally opposite corners coincide as shown below.



Show that the pentagon that results always has an area that is less than three-quarters of the area of the original rectangular piece of paper, regardless of the dimensions of the piece of paper. Justify your answer.

Challenge of the Week # 9 - October 25 to November 1: Each point on the plane represents a plant and each point of the form (m, n)*, with* m *and* n *integers, represents a gardener. Each plant is cared for by the three gardeners who are the closest. (Some plants are equidistant from four gardeners. These are cared for by all four gardeners.) Describe the set of plants that are cared for by the gardener at (0,0).*

Challenge of the Week # 10 - November 1 to November 8: Determine all triples (A, B, C) *of positive integers which satisfy both of the following equations.*

$$\begin{aligned} A + B^2 - C &= 124 \\ A^2 + B - C &= 100 \end{aligned}$$

Challenge of the Week # 11 - November 8 to November 15: There are 1997 planes in space, no two of them parallel, no three containing the same line and no four passing through the same point. These planes divide space into a certain number of regions. There is a line, not in any of the planes, which intersects some of these regions.

a) What is the maximum number of regions that this line can intersect?

a) What is the minimum number of regions that this line can intersect?

Justify your answers.

Challenge of the Week # 12 - November 15 to November 22: Let $A(0,0)$, $B(1,0)$, $D(0,9)$, *and* $C(9,6)$ *be four points. Find the point* P *such that the sum of the distances*

$$PA + PB + PC + PD$$

is minimal. Justify your answer.