Challenges of the Week
Fall Semester 1995-1996

Challenge of the Week # 1 - August 30 to September 8: You have a large container that contains a liter of milk and another large container which contains a liter of coffee. You take a one liter centiliter spoonful of milk out of the milk container and put it into the coffee container. You stir the coffee mixture until it is uniform. Then you take one centiliter of the mixture and put it into the milk. Is there more milk in the coffee container or more coffee in the milk container or are the two amounts the same?

Challenge of the Week # 2 - September 8 to September 15: You have four different letters, some of which weigh two ounces and some of which weigh three ounces. You go to the Post Office and wish to determine how much each letter weighs. At the Post Office, you find a scale. Clearly, you can determine the weight of each letter if you use four weighings and weigh each letter individually. What is the minimum number of weighings necessary to determine the weight of each letter? Justify your answer.

Challenge of the Week # 3 - September 15 to September 22: Suppose

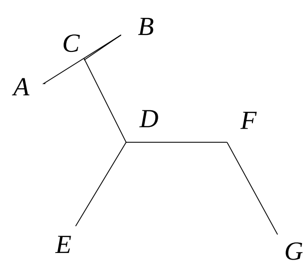
$$x_1, x_2, \ldots, x_{1995}$$

are one thousand nine hundred and ninety five real numbers such that each is equal to the sum of the squares of the other 1994 numbers. Find all possible solutions for $x_1, x_2, \ldots, x_{1995}$.

Challenge of the Week # 4 - September 22 to September 29: Pennies are arranged on a very, very large table (actually, the infinite plane). In the first pattern, each penny touches four other pennies and the straight lines joining the centers of pennies in contact with each other dissect the plane into equal squares. In the second pattern, each penny touches six others and the straight lines joining centers of pennies in contact with each other dissect the plane into equal, equilateral triangles.
Compute the percentage of the plane covered by the pennies for each pattern.

Challenge of the Week # 5 - September 29 to October 5: There are three hands on a large wall clock – an hour hand, a minute hand, and a second hand. A **good time** *is a time when the three hands are all in one-half of the clock face. For example, 1:15:30 is a good time. A* **bad time** *is a time when the three hands are not all in one-half of the clock face. For example, 3:50:30 is a bad time. In a 12 hour period are there more good times than bad times or more bad times than good times or are they equal? Justify your answer.*

Challenge of the Week # 6 - October 6 to October 13: Five toothpicks of equal length are placed in the shape of a 'horse', as shown below. The angle determined by two toothpicks which touch is either 120° *or* 90°. *Prove or disprove the following: It is possible to move one toothpick to another position so that the resulting figure is either congruent to the original 'horse' or is a mirror image of the 'horse'.*



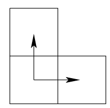
Challenge of the Week # 7 - October 13 to October 20: A square is drawn in the plane. Also in the plane is a point, which I can see, but you cannot see. This point is either inside the square or outside the square. In order to determine which is the case you can draw a sequence of lines on the plane. After you draw each line, I will tell you whether the point is on the line or not and, if not, will tell which side of the line the point is on. What is the minimum number of lines that you must draw in order to be certain that the point is inside the square or be certain that the point is outside the square? Justify your answer.

Challenge of the Week # 8 - October 20 to October 27: If n is a positive integer, let $n^\star$ denote the result of adding to n the sum of its digits. For instance,

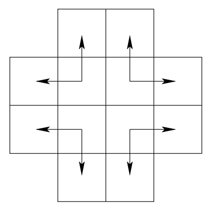
$$5^\star = 10, \qquad 86^\star = 100, \qquad 977^\star = 1000, \qquad 9968^\star = 10000.$$

Find all numbers n with $n^\star = 100,000$. Justify your answer.

Challenge of the Week # 9 - October 27 to November 3: You are given a large number of L-shaped tiles, each consists of three unit squares arranged and marked as follows:



With four of these tiles you can cover a 4×4 checkerboard from which all four corners have been removed (as shown below).



1. *Can a 5×5 checkerboard from which one corner square has been removed be covered by using L-shaped tiles?*
2. *Can a 6×6 checkerboard from which one corner square has been removed be covered?*
3. *Can an 8×8 checkerboard from which one corner square has been removed be covered?*
4. *If n is a positive integer, can a $2^n \times 2^n$ checkerboard from which one corner square has been removed be covered?*

Justify your answers in each case.

Challenge of the Week # 10 - November 3 to November 10: Two rooks, one white and one black, are placed on diagonally opposite corners of an 8×8 chessboard. Both rooks move in the standard way, except neither can move on or across a row or column that is attacked by the other rook. The white rook goes first and play alternates. On each turn a player must move his/her rook - passing is not allowed. The first person who cannot make a legal move on his/her turn loses the game. With the best strategy, who wins the game, the player who plays first or the player who plays second? Justify your answer.