Challenges of the Week
Fall Semester 1994-1995

Challenge of the Week # 1 - September 9 to September 16: Let n be an arbitrary positive integer. Show there is a positive integer x such that $nx + 1$ is not a prime.

Challenge of the Week # 2 - September 16 to September 23: Find the digit in the 1994^{th} place after the decimal in

$$\sqrt{0.999\ldots9},$$

if there are 10,000 nines under the radical. Justify your answer.

Challenge of the Week # 3 - September 23 to September 30: Show that a square can be divided into n smaller squares, if n is any number larger than 5. The smaller squares do not have to be the same size.

Challenge of the Week # 4 - September 30 to October 7: An Egyptian fraction is a fraction of the form $\frac{1}{n}$ where n is a positive integer. The number 1 can be written as the sum of three different Egyptian fractions as

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6}.$$

Show that 1 can be written as the sum of 100 different Egyptian fraction.

Challenge of the Week # 5 - October 7 to October 14: Find all possible values for the following fraction

$$\frac{E \cdot A \cdot S \cdot T \cdot E \cdot R \cdot N}{U \cdot N \cdot I \cdot V \cdot E \cdot R \cdot S \cdot I \cdot T \cdot Y}$$

Each letter represents a digit (0 through 9), distinct letters stand for distinct digits, and a letter has the same value at each occurrence.

Challenge of the Week # 6 - October 14 to October 21: Suppose n is one less than the square of an odd prime. Show that n is a multiple of 8.

Challenge of the Week # 7 - October 21 to October 28: A square cake has frosting on its top and on all four sides. The top is 16" by 16" and the cake is 5" tall. Assume for the sake of this problem that the frosting has zero thickness and that the cake is fat-free.

a) *Show how to cut the cake to serve 8 people so that each person gets exactly the same amount of cake and exactly the same amount of frosting.*

b) *Show how to cut the cake to serve 9 people so that each person gets exactly the same amount of cake and exactly the same amount of frosting.*

Challenge of the Week # 8 - October 28 to November 4: Find all postive integers n such that

$$1! + 2! + 3! + \cdots + n!$$

is the square of an integer.

Challenge of the Week # 9 - November 4 to November 11: The problem of the week for October 14 to October 21 was: If m is an odd prime, show $m^2 - 1$ is a multiple of 8. Several people noticed that this result was true if m was any odd number. The problem for this week:

If m is a prime larger than 3, $m^2 - 1$ is a multiple of 24.

Challenge of the Week # 10 - November 11 to November 18: The sequence $x_1, x_2, \ldots, x_{100}$ is a random rearrangement of the numbers 1 through 100. These numbers are placed around the circumference of a circle Show there are four numbers, consecutively placed on the circle, whose sum exceeds 201.