Challenges of the Week
Fall Semester 2007-2008

Challenge of the Week # 1 - August 24 to August 31: Two cars, A and B, are 100 miles apart, one directly east of the other. Car A moves eastwards towards car B at a rate of 60 mph while car B moves westwards towards car A at a rate of 50 mph. At the same time that the cars begin moving, a very fast hummingbird leaves car A going towards car B at a rate of 80 mph. The hummingbird flies east until it reaches car B and then instantaneously turns around and flies west toward car A. Then, when it reaches car A it instantaneously turns around and flies east toward car B, etc. Eventually the cars and the hummingbird will all meet at a single location at which point they will all instantaneously stop.

1. *What is the total distance flown by the hummingbird?*
2. *What is the total distance that the hummingbird flies in an easterly direction?*

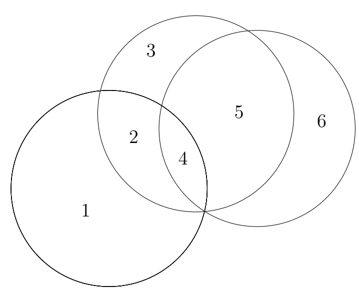
Justify your answer.

Challenge of the Week # 2 - September 7 to September 14: The clock on the wall has two hands, an hour hand and a minute hand. How many times during a 12 hour period is it the case that the two hands are precisely perpendicular? Justify your answer.

Challenge of the Week # 3 - September 14 to September 21: At a junior high dance there are m boys and n girls. Each boy dances with 3 different girls and each girl dances with 5 different boys. There are either 150, 160 or 170 students at the dance. Determine the number of boys and the number of girls at the dance. Justify your answer.

Challenge of the Week # 4 - September 21 to September 28: The clock on the wall has two hands — an hour hand and a minute hand. How many clock positions in a twelve hour time period are there with the property that the result of switching the two hands is also a valid clock position?

Challenge of the Week # 5 - September 28 to October 5: Three identical circles meet at a point. These three circles determine six areas, as illustrated below.



If each of the six areas contains a whole number of square centimeters, prove that

$$area(1) + area(3) + area(6) - area(2) - area(5)$$

is divisible by 3. Justify your answer.

Challenge of the Week # 6 - October 19 to October 26: One hundred line segments of various lengths meet at a common point which is not necessarily the midpoint of any of the line segments. If you pick any three endpoints of the line segments (there are 200 endpoints to choose from), it will be possible to draw a circle through them. In fact, there are a total of 1,313,400 different ways to choose the three points. What is the **minimum** *number of circles that could be determined by this process?*

Challenge of the Week # 7 - October 26 to November 2: Each member of a family drank one cup of coffee, with milk, for breakfast. Some of the cups contained more coffee than milk, some contained more milk than coffee, and some, perhaps, contained the same amount of coffee and milk. Katy's cup contained one-quarter of the total amount of milk and one-sixth of the total amount of coffee. How many members are there in the family and how much milk was in Katy's cup? Justify your answer.

Challenge of the Week # 8 - November 2 to November 9: You have a 29 by 29 grid of squares. You have 29 copies of the numbers 1 through 29. One number is to be placed in each square. The "main" diagonal is the diagonal which goes from the upper left to the lower right and contains 29 squares. The placement of the numbers is such that the sum of the numbers above the main diagonal is exactly three times the sum of the numbers below the main diagonal — the numbers on the main diagonal are not counted in either case. What are the possiblities for the number in the middle square? Justify your answer.

Challenge of the Week # 9 - November 9 to November 16: Eleven friends emailed Christmas greetings. However, each person only emailed three of his or her friends. Could it be possible that every one received an email from the three people to whom they sent emails? Justify your answer.

Challenge of the Week # 10 - November 26 to December 7:

This week's challenge is actually a two-week challenge. It is also the most difficult one given this semester and requires a knowledge of probability. Because no correct solution has been received for the last three challenges, this challenge is worth $80.

Due to security concerns, we do not know how many seats there are on Airforce 1. Let N be the number of seats. The seats are numbered 1 through N. Seat 1 is reserved for the President and seats 2 through N are assigned to reporters. Each reporter is assigned a specific seat. The President gets on the plane, but chooses a seat at random. Then the reporters come on, in order of seat assignment. Each reporter takes his/her assigned seat, if possible. If not possible, the reporter chooses a seat at random. What is the probability that the reporter assigned to seat N will be able to sit in seat N?

(a) *Find this probability when $N = 5$. Justify your answer.*

(b) *Find this probability when $N = 100$. Justify your answer.*