Challenges of the Week
Fall Semester 2006-2007

Challenge of the Week # 1 - August 25 to September 1: Find all positive six digit integers which consist of the digits 1,2,3,4,5,6 and which have the property that no two consecutive digits in the integer are divisible by 2 or 3. Justify your answer.

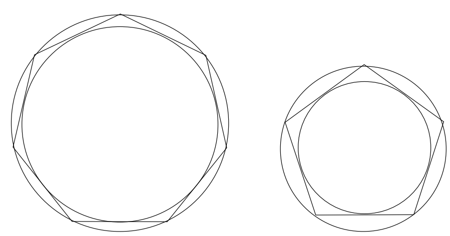
Challenge of the Week # 2 - September 1 to September 15: Is it possible to find fifty-one positive, two-digit integers such that no two of them sum to 100? Justify your answer.

Challenge of the Week # 3 - September 15 to September 22: There are 99 baskets arranged in a circle. Each basket contains pumpkins. Is it possible that the number of pumpkins in every pair of neighboring baskets differ by exactly one? Justify your answer.

Challenge of the Week # 4 - September 22 to September 29:

Find all positive six digit integers which consist of all of the digits 1,2,3,4,5,6 and such that no two consecutive digits **have a sum** *which is divisible by 2 or 3. Justify your answer.*

Challenge of the Week # 5 - September 29 to October 6: A circle is circumscribed about a regular heptagon (a seven-sided polygon) with side length of 2 units and another is inscribed in the same heptagon, as shown below. Also, as shown below, a circle is circumscribed about a regular pentagon with side length of 2 units and another is inscribed in the same pentagon. Which is larger, the area of the region between the two circles determined by the heptagon or the area of the region between the two circles determined by the pentagon? Justify your answer.



Challenge of the Week # 6 - October 9 to October 20: One hundred stones of various weights are arranged in a straight line so that no two neighboring stones differ in weight by more than one ounce. Can the stones be arranged in a circle so that no two neighboring stones differ in weight by more than two ounces? Justify your answer.

Challenge of the Week # 7 - October 20 to October 27: Forty four pigeons are arranged in seventeen cages. Prove that there are five (or more) cages with the same number of pigeons in each of the cages.

Challenge of the Week # 8 - October 27 to November 3: Represent the following sum as a fraction $\frac{a}{b}$ where a and b are relatively prime, positive integers.

$$\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+\cdots+\frac{1}{1+2+3+\cdots+2005+2006}.$$

Challenge of the Week # 9 - November 3 to November 10: Each positive integer is colored either red or blue, but not both. Not all integers have the same color. Further, the product of an integer colored red with an integer colored blue is always an integer colored red. Also, the sum of an integer colored red and an integer colored blue is always an integer colored blue. If two integers are colored red, can you determine the color of their product? If so, what is it? Justify your answer.

Challenge of the Week # 10 - November 10 to November 17: A certain supermarket has a balance scale on which you can weigh your groceries. You put your groceries on one arm of the scale. By placing known weights on the other arm of the scale you can determine the weight of your groceries. This scale is incorrect, however, since one arm is longer than the other.

First, you place your apples on the left arm of the scale and 8 pounds on the other side and it balances. Then, you remove the apples and the weights, place your oranges on the right arm of the scale and 2 pounds on the other side. Again it balances.

Suppose that the actual weight of the apples and the oranges are both integers, the sum of the actual weights is more than 10 pounds and that the oranges actually weigh less than 8 pounds.

What are the actual weights of the apples and the oranges? Justify your answer.