

## Challenges of the Week Fall Semester 2003-2004

Challenge of the Week # 1 - September 5 to September 12: Find all positive integers  $m$  and  $n$  satisfying

$$1! + 2! + \cdots + m! = n^2.$$

Show that your solutions are the only possible solutions.

Challenge of the Week # 2 - September 12 to September 19: Find all positive integers,  $n$ , which have the property that the product of their distinct (positive) divisors is  $n^2$ . An example of such a number is 6 since  $1 \cdot 2 \cdot 3 \cdot 6 = 6^2$ . Justify your answer.

Challenge of the Week # 3 - September 19 to September 26: How many zeros are there at the end of 100!? Justify your answer. (Note: Recall that  $100! = 1 \cdot 2 \cdot 3 \cdots 100$ .)

Challenge of the Week # 4 - September 26 to October 3: Two people go on a hike along the hiking/bike path connecting Charleston and Mattoon. One person leaves Mattoon and goes toward Charleston. At the same time, another person leaves Charleston and goes toward Mattoon. They both walk at a constant, but different, rate. They meet between the cities at 12:00 a. m.. One person gets to Charleston at 4:00 p. m., while the other gets to Mattoon at 9:00 p. m. When did they leave? Justify your answer.

Challenge of the Week # 5 - October 3 to October 10: Compute the following product and explain how you got your result.

$$\left(1 + \frac{3}{3}\right) \left(1 + \frac{3}{4}\right) \left(1 + \frac{3}{5}\right) \cdots \left(1 + \frac{3}{2003}\right).$$

(An **exact** answer, without calculator round-off error, is required.)

Challenge of the Week # 6 - October 10 to October 24: For every integer  $z$ , an integer  $z^*$  is defined. The following equation is true for all integers  $x$  and  $y$ .

$$(xy)^* = (x^* - 1)y^* - x^2 + 1. \quad (\#)$$

Find the values of  $0^*$  and  $1^*$ . Justify your results.

Challenge of the Week # 7 - October 24 to October 31: The huge clock on the courthouse in a certain midwestern city was recently repaired. When the minute and hour hands were put back on the clock, they were put on in the wrong order. As a result, the minute hand moves as fast as the hour hand should and the hour hand moves as fast as the minute hand should. The clock was set at 6:00 to the correct time. A short time later, of course, the clock was not showing the correct time. When is the next time, if ever, that the clock will again show the correct time? Justify your answer.

Challenge of the Week # 8 - October 31 to November 7: For  $x$  a real number, the **fractional part of  $x$** , denoted  $\{x\}$ , is the remainder when the largest integer less than or equal to  $x$  is subtracted away from  $x$ . Thus, we have, for example,

$$\begin{aligned}\{1.75\} &= 0.75, \\ \{132\} &= 0, \\ \{\pi\} &= 0.1415926535897932384 \cdots\end{aligned}$$

Find all solutions to the equation

$$\{5\{4\{3\{2\{x\}\}\}\}\} = x.$$

Challenge of the Week # 9 - November 7 to November 14: You are given 6 congruent balls, two each of the colors red, green, and blue. One ball of each color weighs 16 ounces and the other only 15 ounces. You have a pan balance which will allow you to determine whether two sets of balls have the same weight or to determine which of the two sets is heavier. In only two weighings of the pan balance, describe how to determine which are the 16 ounce balls.

Challenge of the Week # 10 - November 14 to November 21: Four students, no two of the same age, are sitting around a table. They determine the six numbers which are differences in the ages between two of them. Show that 12 divides the product of these differences.